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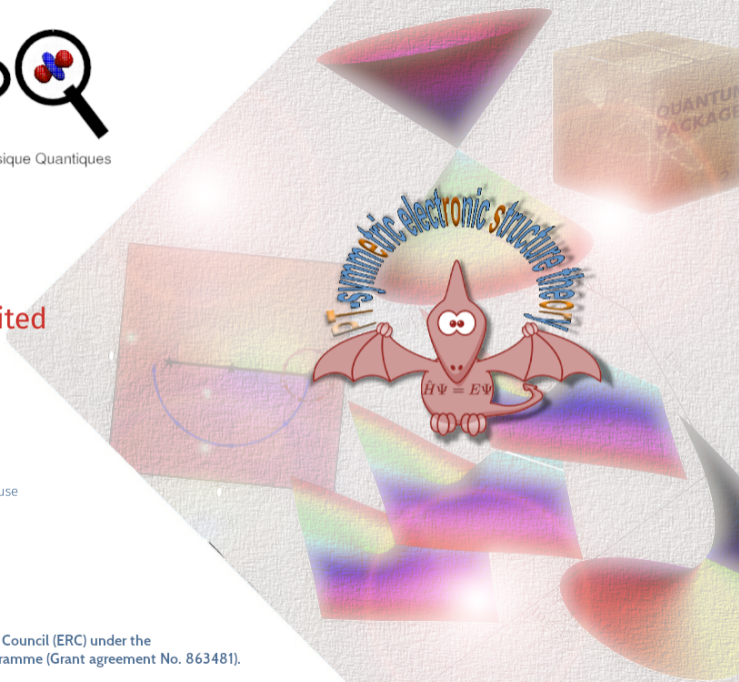
Laboratoire de Chimie et Physique Quantiques

New Electronic Structure Methodologies for Electronic Excited States

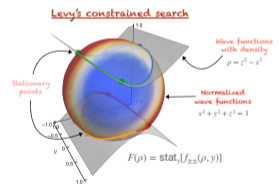
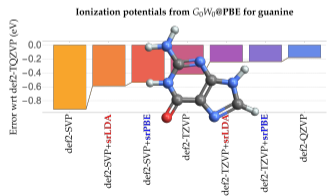
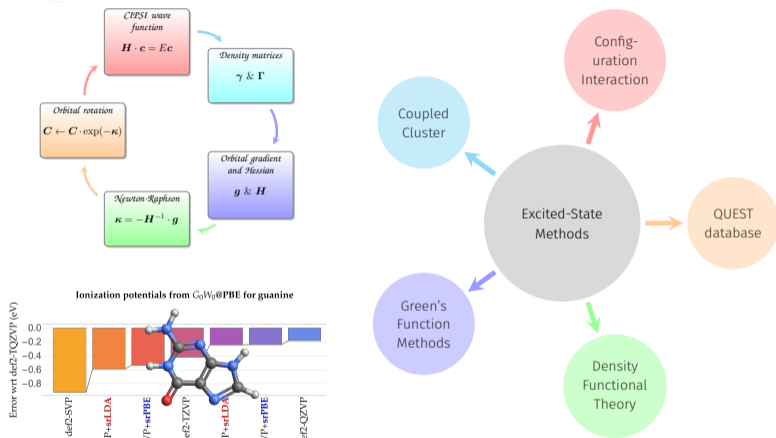
Pierre-François Loos & Friends

12th March 2024

Laboratoire de Chimie et Physique Quantiques, IRSAMC, UPS/CNRS, Toulouse
<https://lcpq.github.io/pterosor>



PTEROSOR has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (Grant agreement No. 863481).



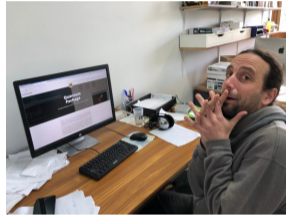
<https://lcpq.github.io/PTEROSOR/>



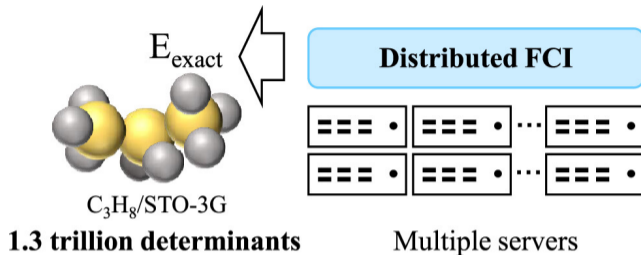
Yann Damour (PhD)



Fábio Kossoski (Postdoc)



Anthony Scemama (Toulouse)



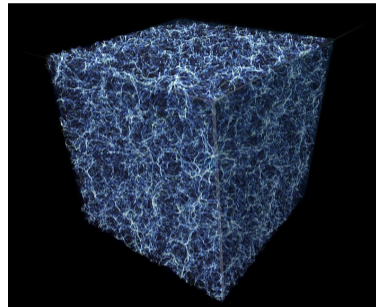
- 🤔 FCI energy of propane (C_3H_8) in STO-3G
- 😬 Active space of 26 electrons in 23 orbitals $\Rightarrow 1.3 \times 10^{12}$ determinants!
- 🙌 512 processes on 256 nodes (40 cores each) for a total wall time of 113.6 hours.
- 🤖 19 TB of memory required!

$$\frac{1}{\sqrt{2}} |\text{🌿}\rangle + \frac{1}{\sqrt{2}} |\text{🍂}\rangle$$



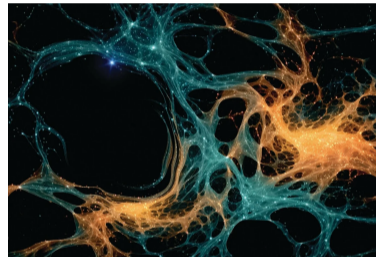
What do we know?

- ▶ Size of Hilbert space increases **exponentially** fast with system size
- ▶ FCI matrix is (very) large but **full of zeros!**
- ▶ Only a tiny fraction of the determinants **significantly contributes** to the energy
- ▶ SCI performs a **sparse exploration** of the FCI space

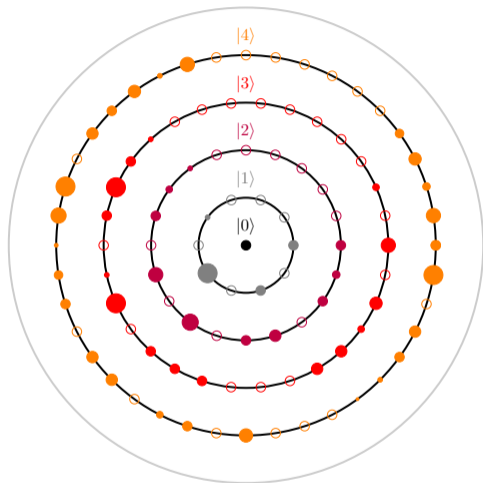


What do we know?

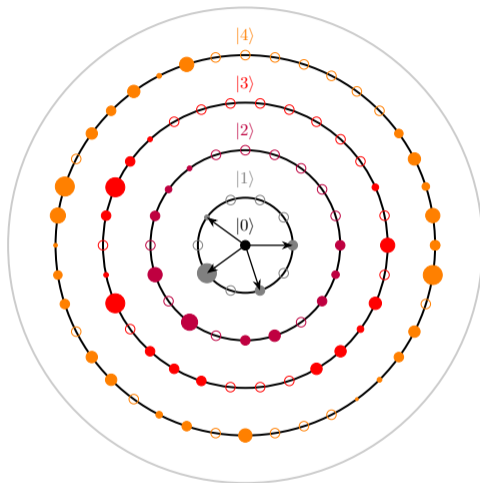
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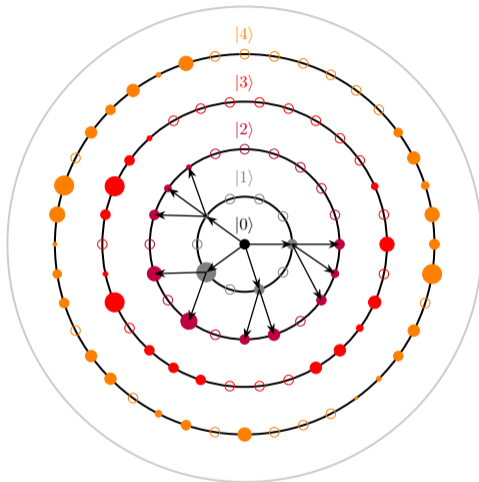
Lemonick, "Cosmic Nothing", Scientific American, 330 (2024) 20



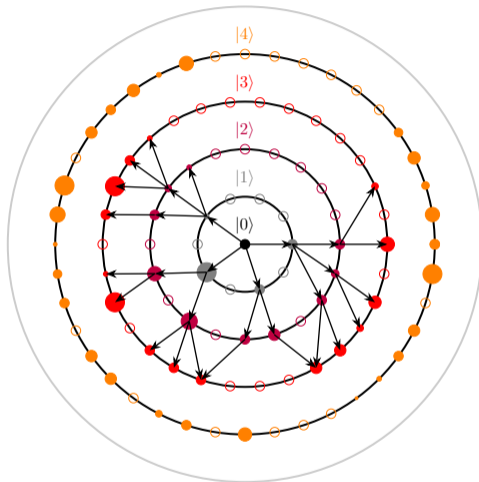
*“Quantum Package 2.0: An Open-Source Determinant-Driven Suite of Programs”,
Garniron et al, JCTC 15 (2019) 3591*



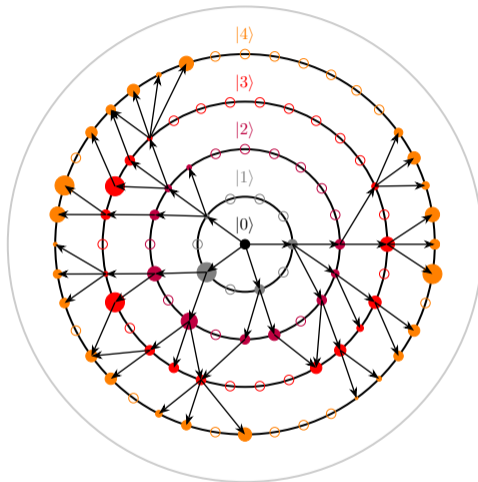
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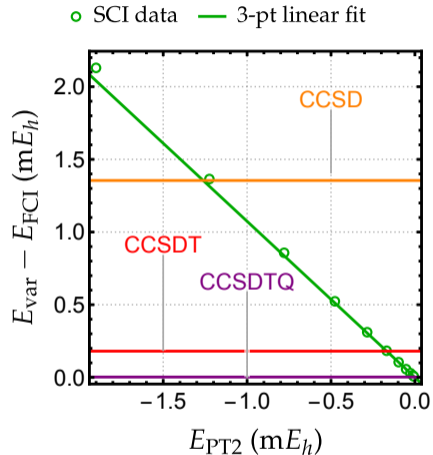


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Method	Energy (E_h)	Error wrt FCI
FCI ¹	-117.100 122 681 461	
CCSD	-117.098 767	1.355 mE_h
CCSD(T)	-117.099 708	0.414 mE_h
CCSDT	-117.099 942 158	0.181 mE_h
CCSDTQ	-117.100 120 230	2.451 μE_h
SCI ²	-117.100 093 52	0.029 mE_h
SCI+PT ³	-117.100 120 66	2.021 μE_h
exFCI ⁴	-117.100 122 89(6)	-0.21(6) μE_h

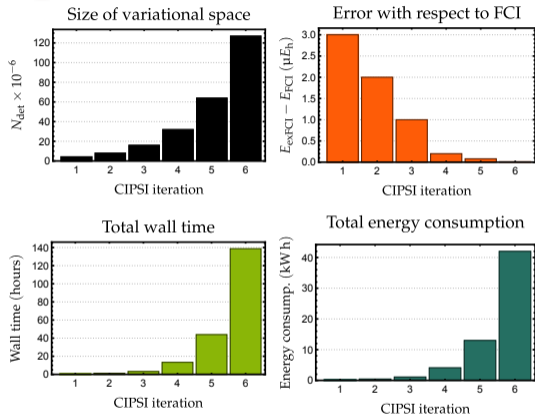


¹Gao et al. JCTC 20 (2024) 1185

²Variational energy obtained with $N_{det} = 32 \times 10^6$

³Perturbatively-corrected variational energy obtained with $N_{det} = 32 \times 10^6$

⁴Extrapolated FCI value obtained via a 3-point linear fit using $N_{det} = 32 \times 10^6$ as the largest variational space



N_{det}	Wall time (hh:mm)	Memory consump.	Energy consump.	Error wrt FCI
2×10^6	00:14	5.3 GB	74 W h	$3 \mu E_h$
4×10^6	00:33	8.1 GB	176 W h	$3 \mu E_h$
8×10^6	01:19	15 GB	438 W h	$2 \mu E_h$
16×10^6	03:12	25 GB	1.1 kW h	$1 \mu E_h$
32×10^6	13:16	47 GB	4.1 kW h	$0.2 \mu E_h$
64×10^6	43:54	83 GB	13 kW h	$0.08 \mu E_h$
127×10^6	138:44	138 GB	42 kW h	$0.01 \mu E_h$

Loos et al, arXiv:2402.13111¹

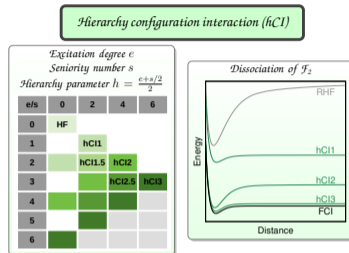
¹Single-node calculation (dual-socket Intel Skylake 6140 CPU@2.3 Ghz with 192 GB of memory and 36 physical CPU cores)

Recent developments

- ▶ Seniority & Hierarchy CI
[Kossoski et al, JPLC 13 (2022) 4342; JCTC 19 (2023) 8654]
- ▶ State-specific CI for excited states
[Kossoski & Loos, JCTC 19 (2023) 2258]
- ▶ State-specific CC for excited states
[Damour et al, arXiv:2401.05048]

Future developments

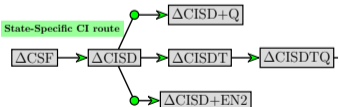
- ▶ Selected CI for resonant states (complex absorbing potential)
[Damour, Kossoski et al, in preparation]



Standard CI route



State-Specific CI route



Standard CC route





Fábris Kossoski (Postdoc)

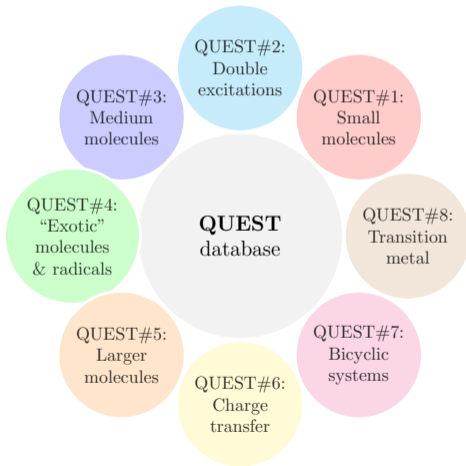


Martial Boggio-Pasqua (Toulouse)



Denis Jacquemin (Nantes)

“The QUEST project aims to provide to the community a large set of highly-accurate excitation energies for various types of excited states”

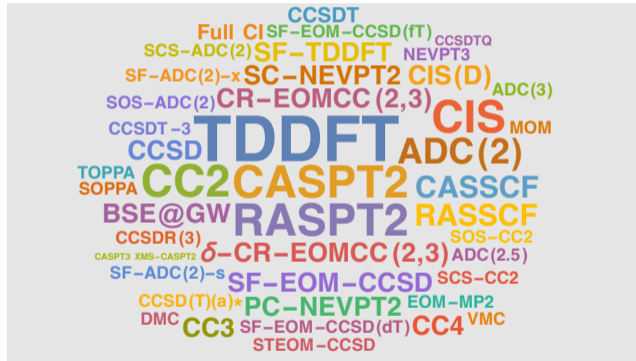


- ▶ #1: JCTC 14 (2018) 4360
- ▶ #2: JCTC 15 (2019) 1939
- ▶ #3: JCTC 16 (2020) 1711
- ▶ #4: JCTC 16 (2020) 3720
- ▶ #5: WIREs 11 (2021) e1517
- ▶ #6: JCTC 17 (2021) 3666
- ▶ #7: JPCA 125 (2021) 10174
- ▶ #8: JCTC 19 (2023) 8782

Zoo of functionals...

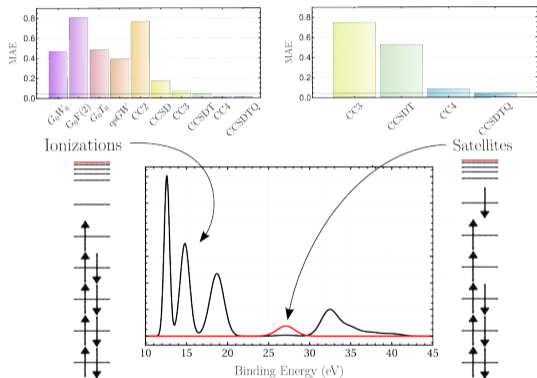


And this is just for excited states...



- ▶ Head-Gordon's group: orbital-optimized DFT for double excitations [JCTC 16 (2020) 1699; JPCL 12 (2021) 4517] and TD-DFT benchmark [JCTC 18 (2022) 3460]
- ▶ Kaupp's group: assessment of hybrid functionals [JCP 155 (2021) 124108]
- ▶ Kallay's and Goerigk's groups: double hybrids [JCTC 15 (2019) 4735; JCTC 17 (2021) 927; JCTC 17 (2021) 5165; JCTC 17 (2021) 4211]
- ▶ Truhlar/Gagliardi's group: p-DFT [JCTC 18 (2022) 6065]
- ▶ Bartlett's group: Variants of EOM-CC for doubly-excited states [JCP 156 (2022) 201102; JPCA 127 (2023) 828; JCP 159 (2023) 094101]
- ▶ Neuscamman's group: QMC for doubly-excited states [JCP 153 (2022) 234105]
- ▶ Filippi's group: QMC for excited states [JCTC 15 (2019) 4889; JCTC 17 (2021) 3426; JCTC 18 (2022) 1089; JCTC 18 (2022) 6722]
- ▶ Gould's group: ensemble DFT [JPCL 13 (2022) 2452]

Antoine's new layer: charged excitations



Marie & Loos, arXiv:2402.13877

Fábris' new layer: double excitations

- ▶ Double excitations are hard!
- ▶ The concept of excitation is relative
- ▶ Improvement of reference data
- ▶ Extension of our previous set

Kossoski et al, in preparation



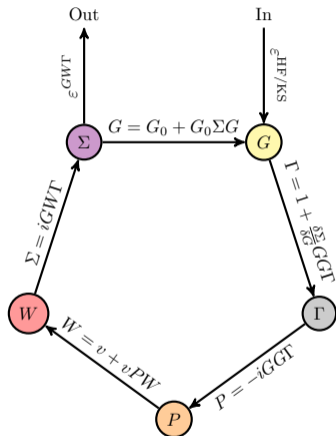
Antoine Marie (PhD)



Abdallah Ammar (Postdoc)



Pina Romaniello (Toulouse)



Hedin, Phys Rev 139 (1965) A796

The wonderful equations of Hedin

$$\underbrace{G(12)}_{\text{Green's function}} = G_0(12) + \int G_0(13) \Sigma(34) G(42) d(34)$$

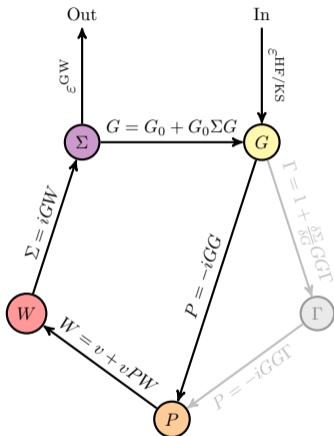
Green's function

$$\underbrace{\Gamma(123)}_{\text{vertex}} = \delta(12)\delta(13) + \int \frac{\delta \Sigma(12)}{\delta G(45)} G(46) G(75) \Gamma(673) d(4567)$$

$$\underbrace{P(12)}_{\text{polarizability}} = -i \int G(13) \Gamma(342) G(41) d(34)$$

$$\underbrace{W(12)}_{\text{screening}} = v(12) + \int v(13) P(34) W(42) d(34)$$

$$\underbrace{\Sigma(12)}_{\text{self-energy}} = i \int G(14) W(13) \Gamma(423) d(34)$$



Hedin, Phys Rev 139 (1965) A796

The GW approximation

$$\underbrace{G(12)}_{\text{Green's function}} = G_0(12) + \int G_0(13) \Sigma(34) G(42) d(34)$$

Green's function

$$\underbrace{\Gamma(123)}_{\text{vertex}} = \delta(12)\delta(13) + \int \frac{\delta \Sigma(12)}{\delta G(45)} \cancel{G(46)G(75)\Gamma(673)} d(4567)$$

vertex

$$\underbrace{P(12)}_{\text{polarizability}} = -i \int \cancel{G(12)\Gamma(342)G(21)} d(34) = -iG(12)G(21)$$

polarizability

$$\underbrace{W(12)}_{\text{screening}} = v(12) + \int v(13) P(34) W(42) d(34)$$

screening

$$\underbrace{\Sigma(12)}_{\text{self-energy}} = i \int \cancel{G(12)W(12)\Gamma(423)} d(34) = iG(12)W(12)$$

self-energy

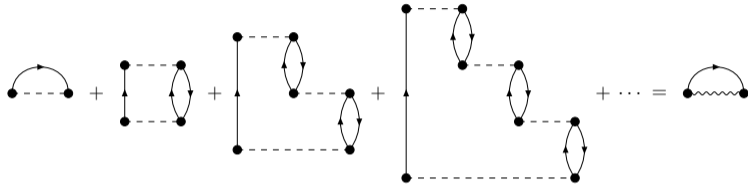
Recent developments

- ▶ Improve self-consistent GW calculations via **similarity renormalization group**
[Marie & Loos, JCTC 19 (2023) 3943]
- ▶ **Connections** between Green's function methods and coupled-cluster theory
[Quintero-Monsebaiz et al, JCP 157 (2022) 231102]

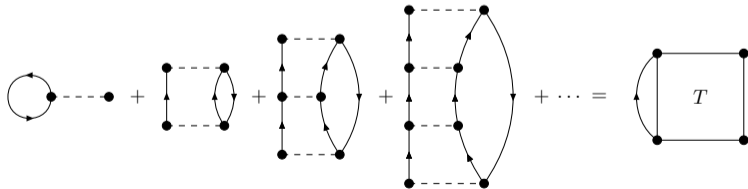
Future developments

- ▶ Hedin's equations and vertex corrections for the **particle-particle channel**
[Marie & Loos, in preparation]
- ▶ Combination of "correlation" channels via **anomalous propagators**
[Marie & Loos, in preparation]
- ▶ **Multireference** version of GW
[Ammar et al, JCP (in press)]

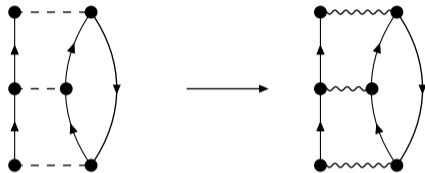
Bubble diagrams of GW



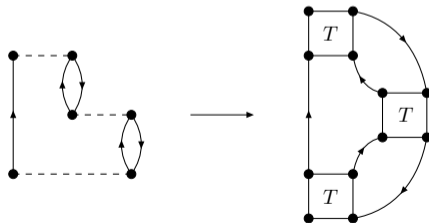
Ladder diagrams of T -matrix



Vertex corrections on top of GW



Vertex corrections on top of T -matrix





Sara Giarrusso (Postdoc)

Levy-Lieb constrained search

$$E = \min_{\rho} \left\{ F[\rho] + \int v(r) \rho(r) dr \right\}$$

Ground-state energy E is minimized over the density ρ . The functional $F[\rho]$ is the Ground-state Functional, and $\int v(r) \rho(r) dr$ is the External potential.

$$F[\rho] = \min_{\substack{\Psi \\ \rho_{\Psi} = \rho}} \langle \Psi | \hat{T} + \hat{V}_{ee} | \Psi \rangle$$

The Ground-state Functional $F[\rho]$ is defined as the minimum energy expectation value over all wavefunctions Ψ that yield the density ρ . The kinetic energy operator \hat{T} is labeled Kinetic, and the electron-electron repulsion operator \hat{V}_{ee} is labeled Repulsion.

Generalized constrained search for excited states

$$E_m = \text{stat}_{\rho} \left\{ F_m[\rho] + \int v(r) \rho(r) dr \right\}$$

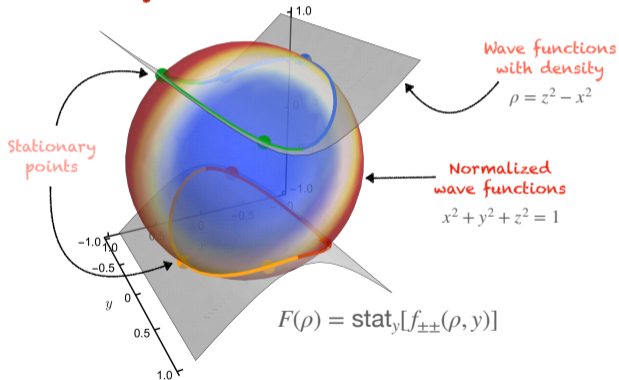
mth excited-state energy → E_m External potential → $v(r)$
Excited-state functional → $F_m[\rho]$

$$F_m[\rho] = \text{stat}_{\substack{\Psi \\ \rho_{\Psi}=\rho}} \langle \Psi | \hat{T} + \hat{V}_{ee} | \Psi \rangle$$

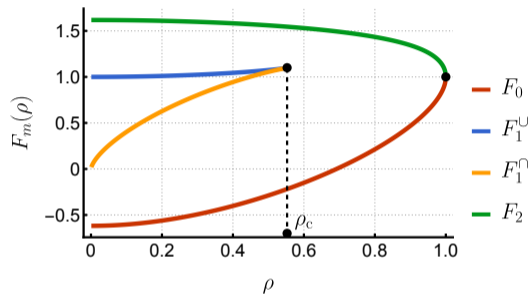
Kinetic → \hat{T} Repulsion → \hat{V}_{ee}

Functional manifold in the asymmetric Hubbard dimer

Levy's constrained search



Exact Functionals of asymmetric Hubbard dimer



Giarrusso & Loos, JPLC 14 (2023) 8780

Future developments

- ▶ Practical Kohn-Sham scheme [Giarrusso & Loos, in preparation]
- ▶ State-specific functionals for realistic systems at the TD-DFT & Δ SCF levels

- ▶ Antoine Marie, Enzo Monino, Roberto Orlando, Yann Damour & Mika Véril
- ▶ Sara Giarrusso, Raúl Quintero-Monsebaiz & Fábris Kossoski
- ▶ Anthony Scemama
- ▶ Denis Jacquemin
- ▶ Martial Boggio-Pasqua
- ▶ Michel Caffarel



https://pfloos.github.io/WEB_LOOS

<https://lcpq.github.io/PTEROSOR>