



European Research Council Established by the European Commission Laboratoire de Chimie et Physique Quantiques

CIPSI: selected configuration interaction methods for ground and excited states

Pierre-François Loos & Friends

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Laboratoire de Chimie et Physique Quantiques, IRSAMC, UPS/CNRS, Toulouse https://lcpq.github.io/pterosor

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General Overview of our Research Group







- This is the oldest and perhaps the easiest method to understand
- The CI wave function is a linear combination of Slater determinants
- CI methods use excited determinants to "improve" the reference (usually Hartree-Fock) wave function



CI is based on the variational principle (like the Hartree-Fock approximation)

Excited Determinants

Excited determinants



CI wave function

$$|\Phi_{CI}\rangle = c_0 |0\rangle + c_S |S\rangle + c_D |D\rangle + c_T |T\rangle + c_Q |Q\rangle + \cdots$$

• When $|S\rangle$ (singles) are taken into account: **CIS**

 $\ket{\Phi_{ ext{CIS}}} = c_0 \ket{0} + c_{ ext{S}} \ket{ ext{S}}$

NB: CIS is an excited state method

• When $|S\rangle$ and $|D\rangle$ are taken into account: **CISD**

 $\ket{\Phi_{ ext{CISD}}} = c_0 \ket{0} + c_{ ext{S}} \ket{ ext{S}} + c_{ ext{D}} \ket{ ext{D}}$

NB: CISD is the most commonly-used CI method

• When $|S\rangle$, $|D\rangle$ and $|T\rangle$ (triples) are taken into account: **CISDT**

 $\left| \Phi_{ ext{CISDT}}
ight
angle = c_{0} \left| 0
ight
angle + c_{ ext{S}} \left| ext{S}
ight
angle + c_{ ext{D}} \left| ext{D}
ight
angle + c_{ ext{T}} \left| ext{T}
ight
angle$

CISDTQ, etc.

When all possible excitations are taken into account, this is called a Full CI calculation (FCI)

 $\left| \Phi_{\text{FCI}} \right\rangle = c_0 \left| 0 \right\rangle + c_{\text{S}} \left| \text{S} \right\rangle + c_{\text{D}} \left| \text{D} \right\rangle + \overline{c_{\text{T}} \left| \text{T} \right\rangle + c_{\text{Q}} \left| \text{Q} \right\rangle + \dots}$

- FCI gives the exact solution of the Schrödinger equation within a given basis
- FCI is becoming more and more fashionable these days (e.g. FCIQMC and CIPSI methods)
- So, why do we care about other methods?
- Because FCI is super computationally expensive!

Pople diagram



"Assume we have 10 electrons in 38 spin MOs: 10 are occupied and 28 are empty"

There is C^k₁₀ possible ways of selecting k electrons out of the 10 occupied orbitals

$$C_n^k = \frac{n!}{k!(n-k)!}$$

▶ There is C_{28}^k ways of distributing them out in the 28 virtual orbitals

- For a given excitation level k, there is $C_{10}^k C_{28}^k$ excited determinants
- The total number of possible excited determinant is

$$\sum_{k=0}^{10} C_{10}^k C_{28}^k = C_{38}^{10} = 472,733,756$$

This is a lot...

For N = 10 and K = 38:

k	Num. of excitations
0	1
1	280
2	17,010
3	393,120
4	4,299,750
5	24,766,560
6	79,115,400
7	142,084,800
8	139,864,725
9	69,069,000
10	13,123,110
Tot.	472,733,756





-----++ ++ ++













S	0	2	4	6









S	0	2	4	6
				sCl6

e/s	0	2	4	6	8
0					
1					
2					
3					
4					
5					
6					

e/s	0	2	4	6	8
0	HF				
1					
2					
3					
4					
5					
6					







e/s	0	2	4	6	8
0					
1					
2					
3					
4					
5					
6					

e/s	0	2	4	6	8
0	sCl0				
1					
2					
3					
4					
5					
6					

e/s	0	2	4	6	8
0					
1		sCI2			
2					
3					
4					
5					
6					





A novel partitioning of the Hilbert space

Hierarchy CI (hCI)

$$h=\frac{e+s/2}{2}$$

- *e*: excitation degree
- ► *s*: seniority number
- ► *h*: hierarchy parameter

e/s	0	2	4	6	8
0					
1					
2					
3					
4					
5					
6					

e/s	0	2	4	6	8
0	HF				
1					
2					
3					
4					
5					
6					

e/s	0	2	4	6	8
0					
1		hCl1			
2					
3					
4					
5					
6					

e/s	0	2	4	6	8
0					
1					
2		hCI1.5			
3					
4					
5					
6					





Hierarchy CI


Excitation-based CI vs Hierarchy CI vs Seniority-based CI







Hierarchy Configuration Interaction





Fábris Kossoski

Kossoski, Damour & Loos, JPCL 13 (2022) 4342 (see also arXiv:2308.14618)

Selected Configuration Interaction: "sparse" exploration of the FCI space

"Among the very large number of determinants contained in the FCI space, only a tiny fraction of them significantly contributes to the energy"

CIPSI = CI using a **Perturbative Selection made Iteratively**

Developed in Toulouse many (many) years ago Huron, Malrieu & Rancurel, JCP 58 (1973) 5745

 Based on old idea by Bender and Davidson, and Whitten and Hackmeyer Bender & Davidson, Phys. Rev. 183 (1969) 23
 Whitten & Hackmeyer, JCP 51 (1969) 5584

 CIPSI (and SCI methods in general) has been recently resurrected! Giner, Scemama & Caffarel, CJC 91 (2013) 879
 Giner, Scemama & Caffarel, JCP 142 (2015) 044115

CIPSI ≈ heat-bath CI (Umrigar) ≈ adaptive sampling CI (Evangelista) ≈ iterative CI (Liu) ≈ incremental CI (Zimmerman) ≈ FCIQMC (Alavi)

CIPSI at the University of Barcelona (1)

Volume 126, number 1

CHEMICAL PHYSICS LETTERS

25 April 1986

CONVERGENCE OF A MULTIREFERENCE SECOND-ORDER MBPT METHOD (CIPSI) USING A ZERO-ORDER WAVEFUNCTION DERIVED FROM AN MS SCF CALCULATION *

J. RUBIO, J.J. NOVOA

Departament de Química Fisica, Facultat de Química, Universitat de Barcelona, Avda. Diagonal 647, 08028 Barcelona, Spain

and

F. ILLAS

Departament de Fisica Química Aplicada, Facultat de Farmacia, Universitat de Barcelona, Avda. Diagonal s/n, 08028 Barcelona, Spain

Received 28 November 1985; in final form 7 February 1986

Approximate natural orbitals and the convergence of a second order multireference many-body perturbation theory (CIPSI) algorithm^{a)}

F. Illas and J. Rubio

Departamento Química Fisica, Facultat de Química, Universitat de Barcelona, C/Marti i Franques 1, 08028-Barcelona, Spain

J. M. Ricart

Departamento Química, Facultat de Química de Tarragona, Universitat de Barcelona, Pca. Imperial Tarraco 1, 43005-Tarrogona, Spain

(Received 6 June 1988; accepted 9 August 1988)

Theor Chim Acta (1992) 82: 229-238

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Treating large intermediate spaces in the CIPSI method through a direct selected CI algorithm

A. Povill, J. Rubio, and F. Illas

Departament de Química Física, Grup de Química Quàntica, Universitat de Barcelona, C/Martí i Franquès 1, E-08028 Barcelona, Spain

Table 1. Summary of results for HF including the dimension of the $\{G\}$, $\{M\}$ and $\{GD\}$ subspaces, the variational MRCI energy, E_{MRCI} , and the error with respect to the FCI energy, ΔE . Energies are in atomic units

$\dim_{\{G\}}$	$\dim \{GD\}$	E _{MRCI}	∆E	
113	81868	- 100.272466	0.001641	
194	125016	-100.272912	0.001195	
278	159009	-100.273067	0.001040	
388	191984	-100.273068	0.001039	
FCI [28]	1.3×10^{7}	-100.274107	-	











- **Green:** reference/variational/internal wave function (zeroth-order or model space)
- **Red:** perturbers or external wave function (first-order or perturbative space)



1. Define a (zeroth-order) *reference* wave function:

$$|\Psi^{(0)}\rangle = \sum_{I \in \mathcal{I}} c_I |I\rangle \qquad \qquad E^{(0)} = \frac{\langle \Psi^{(0)} | H | \Psi^{(0)} \rangle}{\langle \Psi^{(0)} | \Psi^{(0)} \rangle}$$

(-) 0

2. Generate external determinants:

$$\mathcal{A} = \left\{ \left(orall \mathit{l} \in \mathcal{I}
ight) \left(orall \hat{\mathit{T}} \in \mathcal{T}_1 \cup \mathcal{T}_2
ight) : \left| lpha
ight
angle = \hat{\mathit{T}} \left| \mathit{l}
ight
angle
ight\}$$

3. Second-order perturbative contribution of each $|\alpha\rangle$:

$$\delta E(\alpha) = \frac{|\langle \Psi^{(0)} | \hat{H} | \alpha \rangle|^2}{E^{(0)} - \langle \alpha | \hat{H} | \alpha \rangle}$$

- 4. Select $|\alpha\rangle$ with largest $\delta E(\alpha)$ and add them to \mathcal{I}
- 5. Diagonalize \hat{H} in $\mathcal{I} \Rightarrow$ update $|\Psi^{(0)}\rangle$ and $E^{(0)}$
- 6. Iterate

Ground state of Cr₂ in cc-pVQZ: full-valence CAS(28e,198o)



Second-order Epstein-Nesbet correction:

$$E^{(2)} = \sum_{\alpha} \delta E(\alpha)$$

- ► $|\alpha\rangle$'s with largest $\delta E(\alpha)$ have been added to $\Psi^{(0)}$ previously \Rightarrow only small contributions remaining
- \blacktriangleright \mathcal{A} increases with \mathcal{I}
 - \Rightarrow a *very* large number of *very* small contributions
- In practice, we use a semi-stochastic algorithm to compute E⁽²⁾ ⇒ much faster!! Garniron, Scemama, Loos & Caffarel, JCP 147 (2017) 034101
- We linearly extrapolate to $E^{(2)} = 0$ to reach the FCI limit (exFCI)

Ground state of Cr₂ in cc-pVQZ: full-valence CAS(28e,198o)



Garniron et al., JCTC 15 (2019) 3591

The Benzene Blind Challenge: Frozen-core correlation energy (cc-pVDZ)



Eriksen et al. JPCL 11 (2020) 8922

-400-600 $\rightarrow -863 \text{ m}E_h$ $\rightarrow -863 \text{ m}E_h$ $\Delta E_{\text{var.}}$ $\Delta E_{\text{var.}}$ vs E_{PT2} -650 -500 $\rightarrow \Delta E_{\text{var.}} \text{ vs } E_{\text{rPT2}}$ $\rightarrow \Delta E_{\text{var.}} + E_{\text{PT2}}$ Correlation energy (mEh) Correlation energy (mE_h) $\Delta E_{\text{var.}} + E_{\text{rPT2}}$ -700[|] -600 -750 -700 -800 -800 -850 -900L 1000 10^{4} 10 10⁶ 107 108 -200 - 150 - 100 -50 0 Number of determinants $E_{\rm PT2}$ or $E_{\rm rPT2}$ (mE_h)

Loos, Damour & Scemama, JCP 153 (2020) 176101

Performance of CIPSI for C₆H₆/cc-pVDZ (1)

Performance of CIPSI for C₆H₆/cc-pVDZ (2)



Loos, Damour & Scemama, JCP 153 (2020) 176101

Orbital-optimized CIPSI for C₆H₆/cc-pVDZ (and many others)



- Orbital optimization largely accelerates the convergence of selected CI
- Trust-region Newton-Raphson algorithm



Yann Damour

Damour, Véril, Kossoski, Caffarel, Jacquemin, Scemama & Loos, JCP 155 (2020) 176101

Toward a systematic improvement of the fixed-node approximation in diffusion Monte Carlo for solids—A case study in diamond

Cite as: J. Chem. Phys. **153**, 184111 (2020); https://doi.org/10.1063/5.0021036 Submitted: 06 July 2020 . Accepted: 12 October 2020 . Published Online: 11 November 2020

Ib Anouar Benali, Ib Kevin Gasperich, Ib Kenneth D. Jordan, Thomas Applencourt, Ib Ye Luo, Ib M. Chandler Bennett, Ib Jaron T. Krogel, Ib Luke Shulenburger, Ib Paul R. C. Kent, Ib Pierre-François Loos, Ib Anthony Scemama, and Ib Michel Caffarel



Michel Caffarel



"SCI+PT2 methods provide near full CI (FCI) quality quantities with only a small fraction of the determinants of the FCI space"





Anthony Scemama

"Quantum Package 2.0: An Open-Source Determinant-Driven Suite of Programs", Garniron et al., JCTC 15 (2019) 3591 "The aim of the QUEST project is to provide to the community a large set of highly-accurate excitation energies for various types of excited states"





Denis Jacquemin

Zoo of functionals...



And this is just for excited states...





Garniron et al., JCTC 15 (2019) 3591

	$E^{(0)}$		$E^{(0)} + E^{(2)}$		$E^{(0)}+ZE^{(2)}$			
$N_{\rm det}$	GS (a.u.)	ES (a.u.)	GS (a.u.)	ES (a.u.)	$\Delta E(eV)$	GS (a.u.)	ES (a.u.)	$\Delta E(eV)$
28	-149.499574	-149.246268	-150.155(1)	-149.863(1)	7.95(5)	-150.020(1)	-149.743(1)	7.54(5)
58	-149.519908	-149.261390	-150.134(1)	-149.853(1)	7.67(5)	-150.018(1)	-149.744(1)	7.48(5)
131	-149.537424	-149.277496	-150.118(1)	-149.8427(9)	7.52(4)	-150.017(1)	-149.7449(9)	7.39(4)
268	-149.559465	-149.298484	-150.1035(9)	-149.8308(9)	7.42(4)	-150.0158(9)	-149.7457(9)	7.35(4)
541	-149.593434	-149.323302	-150.0845(8)	-149.8186(8)	7.24(4)	-150.0152(8)	-149.7463(8)	7.32(4)
1101	-149.627202	-149.354807	-150.0683(8)	-149.8045(8)	7.18(3)	-150.0137(8)	-149.7460(8)	7.28(3)
2207	-149.663850	-149.399522	-150.0549(7)	-149.7879(7)	7.26(3)	-150.0132(7)	-149.7462(7)	7.27(3)
4417	-149.714222	-149.448133	-150.0409(6)	-149.7762(6)	7.20(3)	-150.0130(6)	-149.7478(6)	7.22(3)
8838	-149.765886	-149.496401	-150.0296(5)	-149.7655(5)	7.19(2)	-150.0124(5)	-149.7473(5)	7.21(2)
17 680	-149.817301	-149.545048	-150.0239(4)	-149.7615(4)	7.14(2)	-150.0141(4)	-149.7505(4)	7.17(2)
35 380	-149.859737	-149.587668	-150.0216(3)	-149.7582(3)	7.17(1)	-150.0161(3)	-149.7518(3)	7.19(1)
70 764	-149.893273	-149.623235	-150.0207(2)	-149.7566(3)	7.18(1)	-150.0174(2)	-149.7530(3)	7.19(1)
141 545	-149.919463	-149.650109	-150.0214(2)	-149.7572(2)	7.189(8)	-150.0194(2)	-149.7550(2)	7.196(8)
283 108	-149.937839	-149.669735	-150.0224(2)	-149.7576(2)	7.206(7)	-150.0211(2)	-149.7562(2)	7.209(7)
566 226	-149.950918	-149.683278	-150.0233(1)	-149.7580(1)	7.217(6)	-150.0223(1)	-149.7570(1)	7.219(6)
1 132 520	-149.960276	-149.693053	-150.0238(1)	-149.7588(1)	7.212(5)	-150.0231(1)	-149.7580(1)	7.214(5)
2 264 948	-149.968203	-149.700907	-150.0240(1)	-149.7590(1)	7.211(4)	-150.0235(1)	-149.7584(1)	7.212(4)
4 529 574	-149.975230	-149.708061	-150.0245(1)	-149.7594(1)	7.215(4)	-150.0241(1)	-149.7589(1)	7.216(4)
9 057 914	-149.981770	-149.714526	-150.02463(9)	-149.75981(8)	7.206(3)	-150.02434(9)	-149.75948(8)	7.207(3)
18 110 742	-149.987928	-149.720648	-150.02495(7)	-149.76025(8)	7.203(3)	-150.02474(7)	-149.76000(8)	7.204(3)
36 146 730	-149.993593	-149.726253	-150.02527(6)	-149.76065(7)	7.198(3)	-150.02502(6)	-149.760 47(7)	7.198(3)

Table 1. Zeroth-Order Energy $E^{(0)}$, Second-Order Perturbative Correction $E^{(2)}$, and Its Renormalized Version $ZE^{(2)}$ (in hartree) of CN3 for Increasingly Large Wave Functions^a

^aThe excitation energy ΔE (in eV) is the energy difference between the ground state (GS) and the excited state (ES). The statistical error, corresponding to one standard deviation, is reported in parentheses.

QUEST #1 – small molecules –

JCTC Cornal of Chemical Theory and Computati

S Cite This: J. Chem. Theory Comput. 2018, 14, 4360-4379

pubs.acs.org/JCT0

A Mountaineering Strategy to Excited States: Highly Accurate Reference Energies and Benchmarks

Pierre-François Loos,* $^{\dagger \odot}$ Anthony Scemama,[†] Aymeric Blondel,[‡] Yann Garniron,[†] Michel Caffarel,[†] and Denis Jacquemin* $^{\dagger \odot}$

¹Laboratoire de Chimie et Physique Quantiques, Université de Toulouse, CNRS, UPS, 31013 Toulouse Cedex 6, France ²Laboratoire CEISAM - UMR CNRS 6230, Université de Nantes, 2 Rue de la Houssinière, BP 92208, 44322 Nantes Cedex 3, France



- 110 vertical excitation energies (VTEs) and oscillator strengths
- 18 small molecules from 1 to 3 non-H atoms
- CC3/aug-cc-pVTZ geometries
- mostly singly-excited states and very few doubly-excited states
- rely on FCI to define "theoretical best estimates" (TBEs)
- aug-cc-pVTZ and CBS vertical energies
- benchmark popular excited-state methods accounting for double and triple excitations

Benchmarking excited-state methods vs TBE/aug-cc-pVTZ



Errors in ADC(2) & ADC(3) for states with large (> 0.15 eV) ADC(2) error



See also Loos & Jacquemin, JPCL 11 (2020) 974

QUEST #2 - double excitations -

Article
Chemical Theory and Computation______Article
Chemical Theory Comput. 2019, 15, 1939–1956 pubSacs.org/JCTC

Reference Energies for Double Excitations

Pierre-François Loos,**[†][©] Martial Boggio-Pasqua,[†][©] Anthony Scemama,[†] Michel Caffarel,[†] and Denis Jacquemin[‡][©]

[†]Laboratoire de Chimie et Physique Quantiques (UMR 5626), Université de Toulouse, CNRS, UPS, 31062 Toulouse, France [‡]Laboratoire CEISAM (UMR 6230), CNRS, Université de Nantes, 44399 Cedex 3 Nantes, France



20 VTEs for doubly-excited states

- 14 small- and medium-sized molecules
- mostly rely on FCI to define TBEs (except for the largest molecules)
- aug-cc-pVTZ and CBS vertical energies
- benchmark excited-state methods including at least triple excitations
- additional benchmarks of multi-configurational methods

Reference energies for double excitations



QUEST #3 - medium-sized molecules -



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Article

A Mountaineering Strategy to Excited States: Highly Accurate Energies and Benchmarks for Medium Sized Molecules

Pierre-François Loos,
* Filippo Lipparini, * Martial Boggio-Pasqua, Anthony Scemama, and Deni
s Jacquemin *

Medium-size molecules



- 238 VTEs (and oscillator strengths) with mostly singly-excited states and aug-cc-pVTZ basis
- 27 small- and medium-sized molecules with 4 to 6 non-H atoms
- rely mostly on CCSDT or CCSDTQ to define TBEs
- benchmark popular excited-state methods accounting for double and triple excitations
- recently improved TBEs with CC4 and CCSDTQ [JCP 154 (2021) 221103; JCTC 18 (2022) 4418]

QUEST #4 - "exotic" molecules and radicals -



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Mountaineering Strategy to Excited States: Highly Accurate Energies and Benchmarks for Exotic Molecules and Radicals

Pierre-François Loos,* Anthony Scemama, Martial Boggio-Pasqua, and Denis Jacquemin*



 two subsets of excitations and oscillator strengths

Article

- an "exotic" subset of 30 VTEs for closed-shell molecules containing F, Cl, P, and Si
- a "radical" subset of 51 doublet-doublet transitions in 24 small radicals
- total of 81 TBEs mostly obtained at the FCI/aug-cc-pVTZ level
- benchmark popular excited-state methods (U vs RO)

QUEST #5 - larger molecules -

Received: 2 December 2020 Revised: 5 January 2021 Accepted	ed: 7 January 2021		
DOI: 10.1002/wcms.1517			12 nour out
FOCUS ARTICLE	WIREs COMPUTATIONAL MOLECULAR SCIENCE	WILEY	13 new syst
TOCOSARTICEE	-		molecules a

QUESTDB: A database of highly accurate excitation energies for the electronic structure community

Mickaël Véril¹ | Anthony Scemama¹[©] | Michel Caffarel¹ | Filippo Lipparini² | Martial Boggio-Pasqua¹ | Denis Jacquemin³ | Pierre-François Loos¹[©]

- 13 new systems composed by small molecules as well as larger molecules
- 80 new transitions the vast majority being of CCSDT quality
- benchmark popular excited-state methods over the entire database
Single- vs Multi-reference methods



JCTC 18 (2022) 2418; JCP 157 (2022) 014103

The QUEST website





Mika Véril

Véril et al. WIREs Comput. Mol. Sci. 11 (2021) e1517

 $https://lcpq.github.io/QUESTDB_website$

- Head-Gordon's group: orbital-optimized DFT for double excitations [JCTC 16 (2020) 1699; JPCL 12 (2021)
 4517] and TD-DFT benchmark [JCTC 18 (2022) 3460]
- Kaupp's group: assessment of hybrid functionals [JCP 155 (2021) 124108]
- Kallay's and Goerigk's groups: double hybrids [JCTC 15 (2019) 4735; JCTC 17 (2021) 927; JCTC 17 (2021) 5165; JCTC 17 (2021) 4211]
- Truhlar/Gagliardi's group: p-DFT [JCTC 18 (2022) 6065]
- Bartlett's group: Variants of EOM-CC for doubly-excited states [JCP 156 (2022) 201102; JPCA 127 (2023) 828; JCP 159 (2023) 094101]
- Neuscamman's group: QMC for doubly-excited states [JCP 153 (2022) 234105]
- Filippi/Scemama's groups: QMC for excited states [JCTC 15 (2019) 4889; JCTC 17 (2021) 3426; JCTC 18 (2022) 1089; JCTC 18 (2022) 6722]
- Gould's group: ensemble DFT [JPCL 13 (2022) 2452]

- Antoine Marie, Enzo Monino, Roberto Orlando, Yann Damour & Mika Véril
- Sara Giarrusso, Raúl Quintero-Monsebaiz & Fábris Kossoski
- Anthony Scemama
- Denis Jacquemin
- Martial Boggio-Pasqua
- Michel Caffarel





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https://pfloos.github.io/WEB_LOOS

https://lcpq.github.io/PTEROSOR