

# Higher roots of the Schrödinger equation

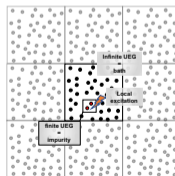
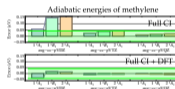
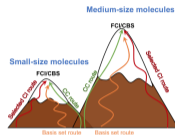
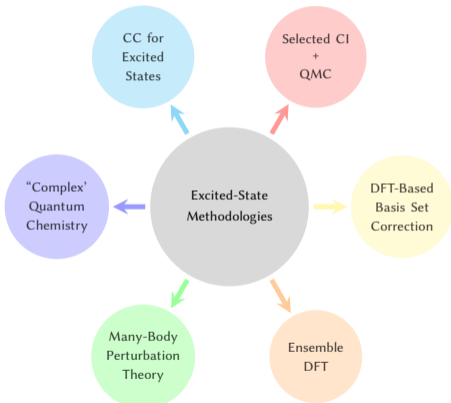
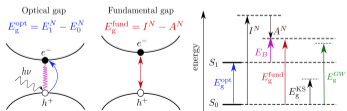
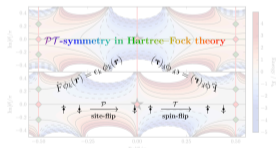
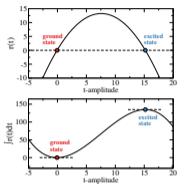
Pierre-François (Titou) LOOS & Friends

Laboratoire de Chimie et Physique Quantiques (UMR 5626),  
Université de Toulouse, CNRS, UPS, Toulouse, France.

NanoX-FeRMI days (March 10th 2022)

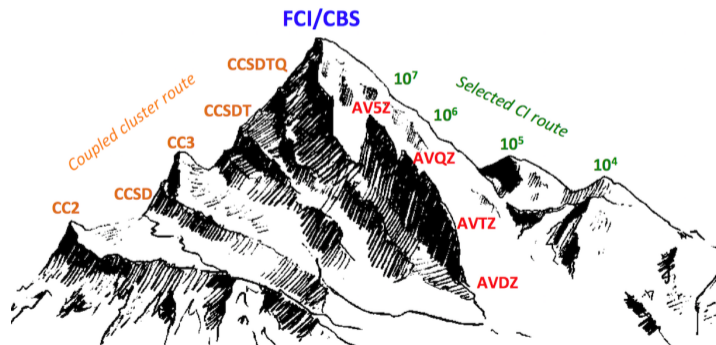


# General overview of our research group



## Section 1

## Selected CI



## Selected CI or how to create new methods with new acronyms

*“SCI+PT2 methods provide near full CI (FCI) quality energies with only a small fraction of the determinants of the FCI space”*

- CIPSI (Malrieu, Evangelisti, Angeli, Spiegelman, Giner, Caffarel, Scemama, etc)
- Semistochastic Heat-bath CI (Sharma & Umrigar)
- Adaptive sampling CI (Evangelista & Tubman)
- Incremental CI (Zimmerman)
- Iterative CI (Liu & Hoffmann)
- FCIQMC (Alavi & Booth)
- ...

# One selected CI (SCI) algorithm to rule them all

## CIPSI = CI using a Perturbative Selection made Iteratively

- Developed in Toulouse many (many) years ago  
Huron, Malrieu & Rancurel, JCP 58 (1973) 5745
- Based on old idea by Bender and Davidson, and Whitten and Hackmeyer  
Bender & Davidson, Phys. Rev. 183 (1969) 23  
Whitten & Hackmeyer, JCP 51 (1969) 5584
- CIPSI (and SCI methods in general) has been recently resurrected!  
Giner, Scemama & Caffarel, CJC 91 (2013) 879  
Giner, Scemama & Caffarel, JCP 142 (2015) 044115
- CIPSI  $\approx$  deterministic version of FCIQMC  
Caffarel et al., Recent Progress in Quantum Monte Carlo (2016) Chap. 2, 15-46.

## Selected CI methods

*“SCI+PT2 methods provide near full CI (FCI) quality energies with only a small fraction of the determinants of the FCI space”*



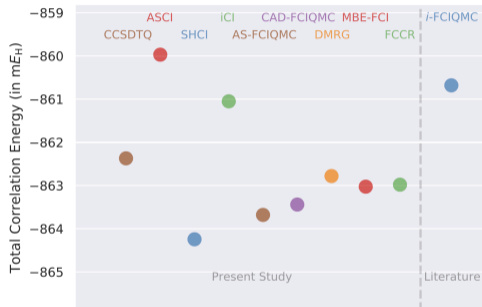
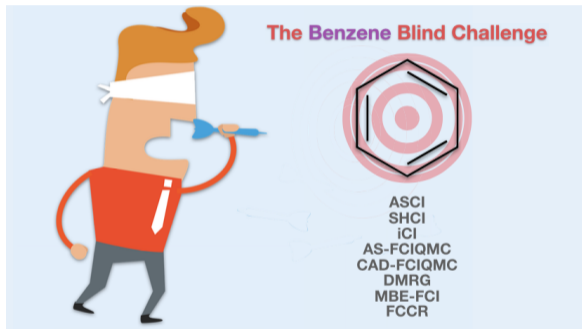
Anthony Scemama



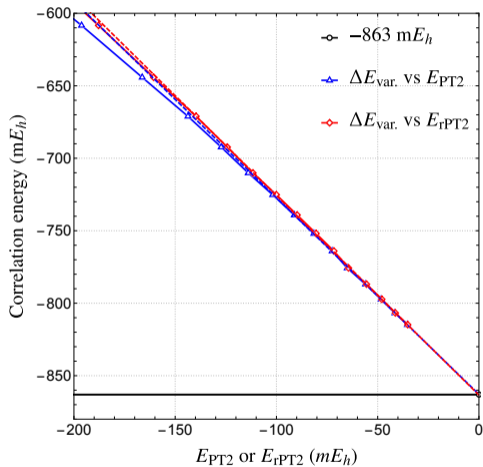
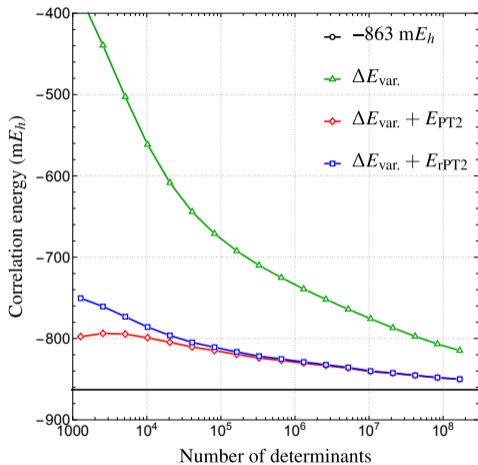
Michel Caffarel

*“Quantum Package 2.0: An Open-Source Determinant-Driven Suite of Programs”,  
Garniron et al., JCTC 15 (2019) 3591*

# The Benzene Blind Challenge: Frozen-core correlation energy (cc-pVDZ)

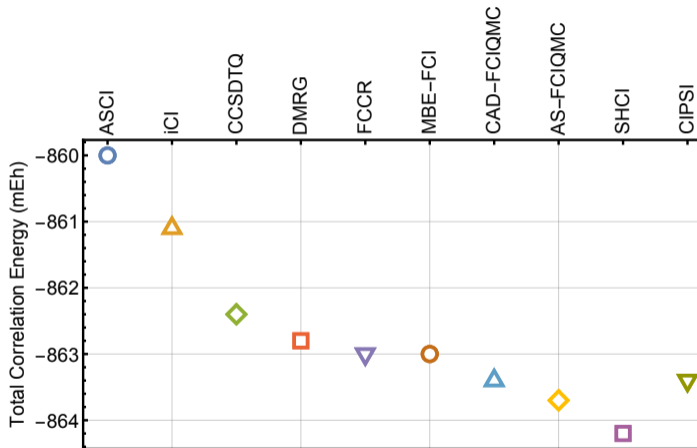


Eriksen et al. JPCL 11 (2020) 8922

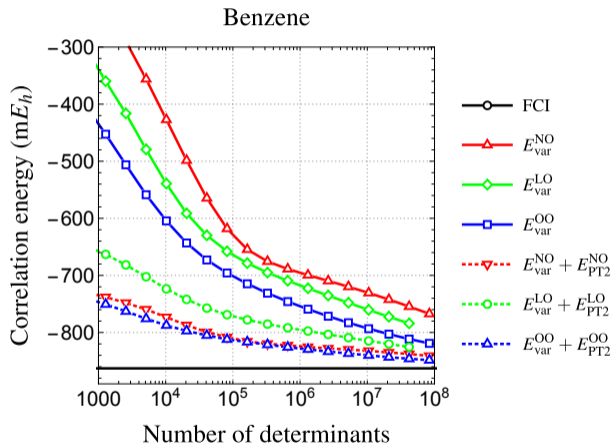
Performance of CIPSI for  $C_6H_6/cc\text{-pVDZ}$  (1)

Loos, Damour &amp; Scemama JCP 153 (2020) 176101



Performance of CIPSI for  $C_6H_6/cc\text{-pVDZ}$  (2)

Loos, Damour & Scemama JCP 153 (2020) 176101

Orbital-optimized CIPSI for  $C_6H_6/cc\text{-pVDZ}$  (and many others)Yann Damour (PhD) ⇒ **POSTER**

Damour, Veril, Kossoski, Caffarel, Jacquemin, Scemama & Loos JCP 155 (2020) 176101

## CIPSI trial wave functions for periodic solids

# Toward a systematic improvement of the fixed-node approximation in diffusion Monte Carlo for solids—A case study in diamond

Cite as: J. Chem. Phys. **153**, 184111 (2020); <https://doi.org/10.1063/5.0021036>

Submitted: 06 July 2020 . Accepted: 12 October 2020 . Published Online: 11 November 2020

 Anouar Benali,  Kevin Gasperich,  Kenneth D. Jordan, Thomas Applencourt,  Ye Luo,  M. Chandler Bennett,  Jaron T. Krogel,  Luke Shulenburger,  Paul R. C. Kent,  Pierre-François Loos,  Anthony Scemama, and  Michel Caffarel

See also [Scemama, Giner, Benali & Loos JCP 153 \(2021\) 174107](#) for a range-separated approach in molecules

# Highly-accurate excitation energies: The QUEST project (1)

*“The aim of the QUEST project is to provide to the community a large set of highly-accurate excitation energies for various types of excited states”*

QUEST#2:  
Double  
excitations

QUEST#1:  
Small  
molecules

QUEST#3:  
Medium  
molecules

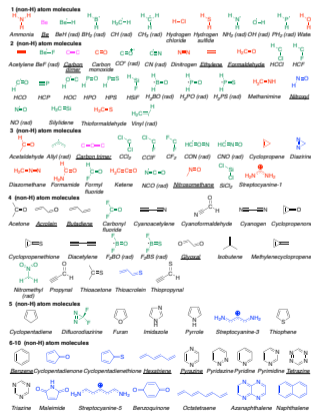
QUEST  
database

QUEST#7:  
Bicyclic  
systems

QUEST#4:  
“Exotic”  
molecules  
& radicals

QUEST#5:  
Larger  
molecules

QUEST#6:  
Charge  
transfer



Mika Veril (PhD)



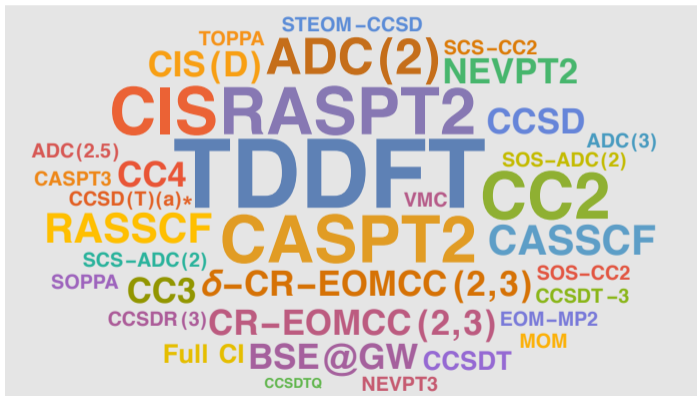
Martial Boggio-Pasqua



Denis Jacquemin

# Electronic structure nightmare...

And this is just for excited states...



# Highly-accurate excitation energies: The QUEST project (2)

QUEST: a database of highly-accurate excitation energies

HOME DATASET SUBSETS REFERENCES

AVTZ AVDZ

NEVPT2 CASPT2 CCSDTQ

GROUND STATE

QUESTDB

Medium-size molecules

QUantum Excited States DataBase

FCI/CBS Selected

Small-size molecules

UCCSDTQ Full CI CIPSI

UCCSDT

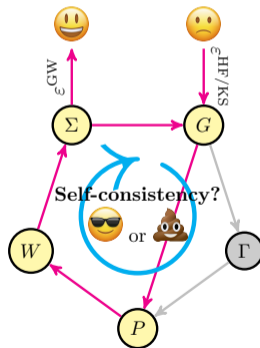
A mountaineering strategy to excited states

Véril et al. WIREs Comput. Mol. Sci. 11 (2021) e1517

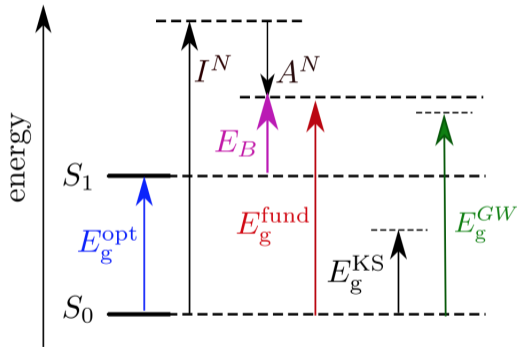
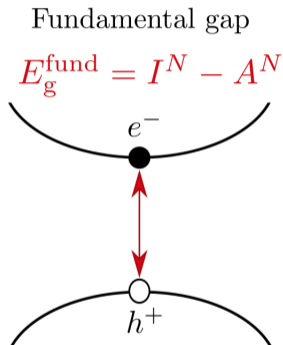
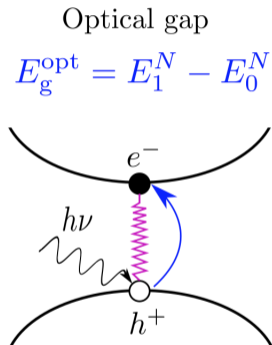
[https://lcpq.github.io/QUESTDB\\_website/](https://lcpq.github.io/QUESTDB_website/)

## Section 2

## Many-body perturbation theory



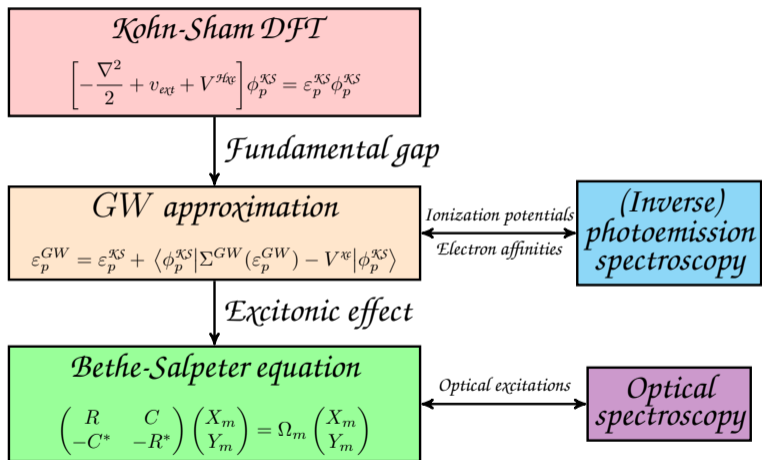
## Fundamental gap vs Optical gap



Blase, Duchemin, Jacquemin &amp; Loos JPCL 11 (2020) 7371

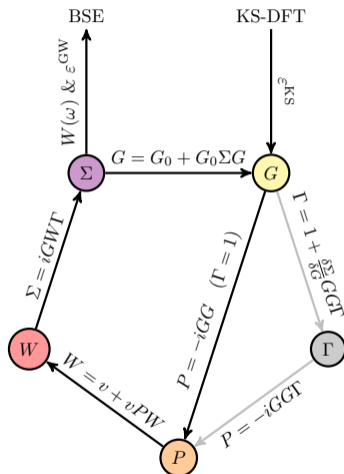


## The MBPT chain of actions



Blase, Duchemin, Jacquemin &amp; Loos JPCL 11 (2020) 7371

## The GW approximation: Hedin's pentagon

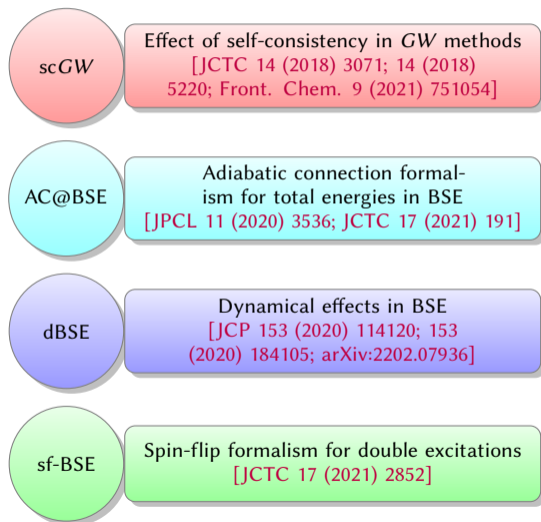


Hedin, Phys. Rev. 139 (1965) A796

# The bridge between TD-DFT and BSE

TD-DFT	Connection	BSE
<p>One-point density</p> $\rho(1)$	$\rho(1) = -iG(11^+)$	<p>Two-point Green's function</p> $G(12)$
<p>Two-point susceptibility</p> $\chi(12) = \frac{\partial \rho(1)}{\partial U(2)}$	$\chi(12) = -iL(12; 1^+2^+)$	<p>Four-point susceptibility</p> $L(12; 34) = \frac{\partial G(13)}{\partial U(42)}$
<p>Two-point kernel</p> $K(12) = v(12) + \frac{\partial V^{xc}(1)}{\partial \rho(2)}$		<p>Four-point kernel</p> $i\Xi(1234) = v(13)\delta(12)\delta(34) - \frac{\partial \Sigma^{xc}(12)}{\partial G(34)}$

## Recent Developments in Many-Body Perturbation Theory [JPCL 11 (2020) 7371]



Stefano Di Sabatino (Postdoc NanoX)

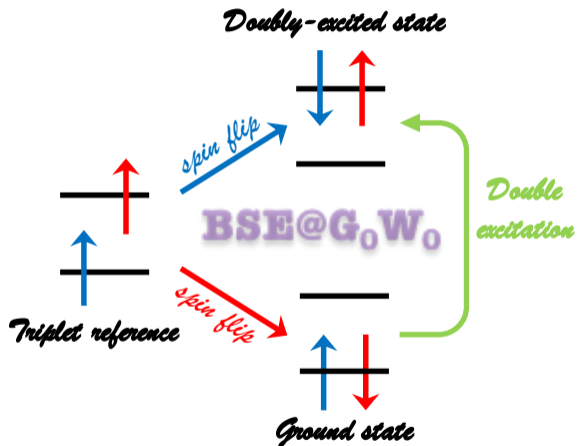


Roberto Orlando (PhD 80|PRIME)



Pina Romaniello (LPT)

## Spin-flip BSE formalism



Enzo Monino (PhD ERC)

Monino &amp; Loos, JCTC 17 (2021) 2852

## Bethe-Salpeter for ground-state energies



pubs.acs.org/JPCL

Letter

# Pros and Cons of the Bethe–Salpeter Formalism for Ground-State Energies

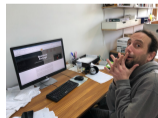
Pierre-François Loos,\* Anthony Scemama, Ivan Duchemin, Denis Jacquemin,\* and Xavier Blase\*



Cite This: *J. Phys. Chem. Lett.* 2020, 11, 3536–3545

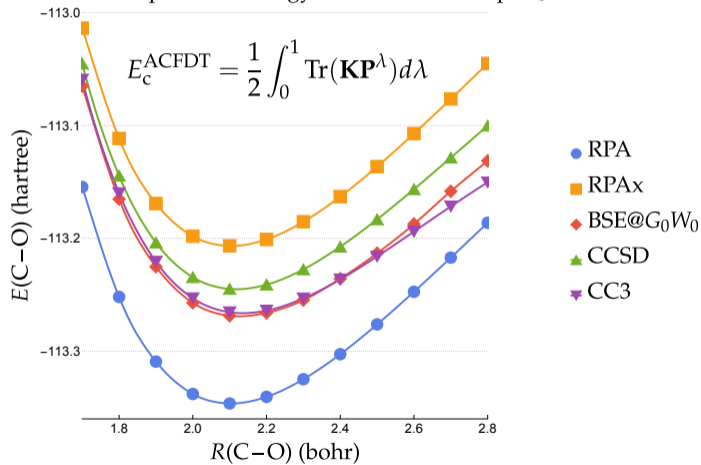


Read Online

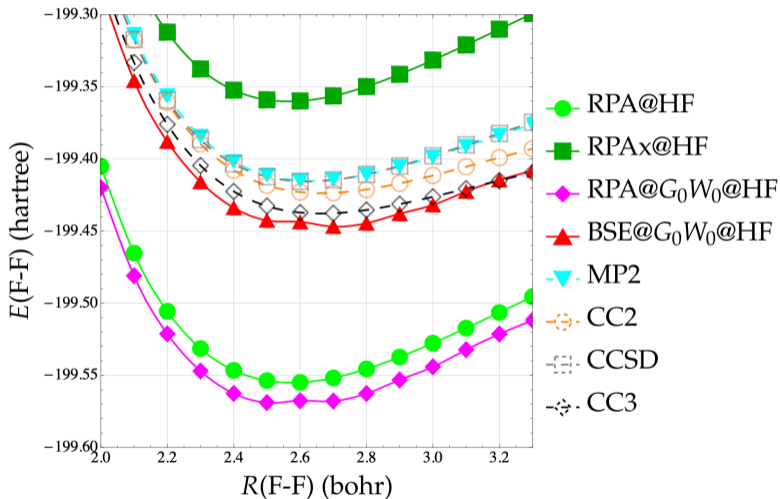


## Bethe-Salpeter for ground-state energies

Ground-state potential energy surface of CO/cc-pVQZ



## Bethe-Salpeter for ground-state energies





## The elephant in the room of GW

# Green Functions and Self-Consistency: Insights From the Spherium Model

Pierre-François Loos,<sup>\*,†</sup> Pina Romaniello,<sup>‡,¶</sup> and J. A. Berger<sup>†,¶</sup>

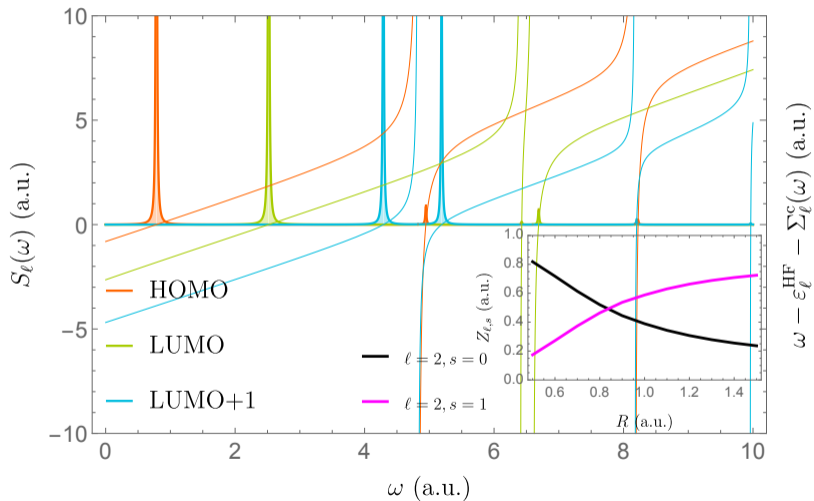
<sup>†</sup>Laboratoire de Chimie et Physique Quantiques, Université de Toulouse, CNRS, UPS, 31062 Toulouse, France

<sup>‡</sup>Laboratoire de Physique Théorique, Université de Toulouse, CNRS, UPS, 31062 Toulouse, France

<sup>¶</sup>European Theoretical Spectroscopy Facility (ETSF)



## The elephant in the room of GW



# The elephant in the room of GW

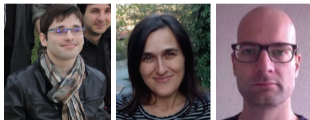
# JCTC

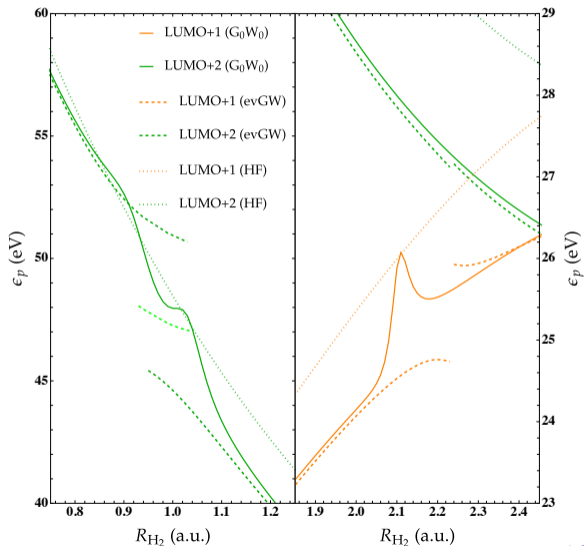
Journal of Chemical Theory and Computation

Article

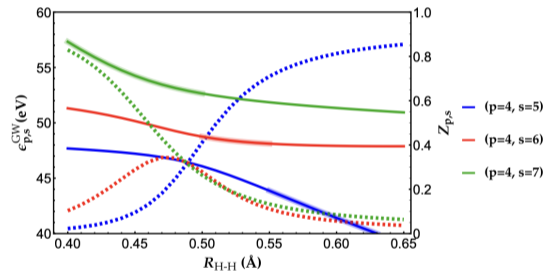
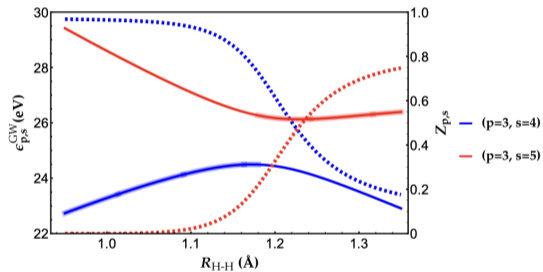
Cite This: *J. Chem. Theory Comput.* 2018, 14, 5220–5228[pubs.acs.org/JCTC](https://pubs.acs.org/JCTC)

## Unphysical Discontinuities in GW Methods

Mickaël Vériel,<sup>†</sup> Pina Romaniello,<sup>‡,¶</sup> J. A. Berger,<sup>†,¶</sup> and Pierre-François Loos<sup>\*,†,¶</sup><sup>†</sup>Laboratoire de Chimie et Physique Quantiques, <sup>‡</sup>Laboratoire de Physique Théorique, and <sup>¶</sup>European Theoretical Spectroscopy Facility (ETSF), Université de Toulouse, CNRS, UPS, Toulouse, France

The elephant in the room of GW (H<sub>2</sub>/6-31G)

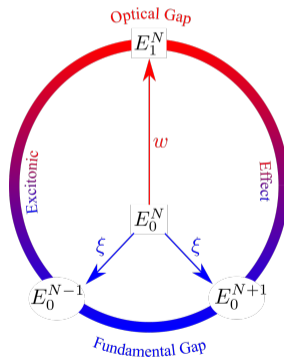
## Intruder States &amp; Regularization



Monino & Loos, arXiv:2202.11589

## Section 3

## Density-functional theory for ensembles



© Bruno Senjean

## eDFT for neutral excitations

# A weight-dependent local correlation density-functional approximation for ensembles <sup>EP</sup>

Cite as: *J. Chem. Phys.* **152**, 214101 (2020); doi: [10.1063/5.0007388](https://doi.org/10.1063/5.0007388)  
 Submitted: 12 March 2020 • Accepted: 12 May 2020 •  
 Published Online: 1 June 2020



Pierre-François Loos <sup>1,a)</sup> and Emmanuel Fromager <sup>2,b)</sup>

## AFFILIATIONS

<sup>1</sup>Laboratoire de Chimie et Physique Quantiques (UMR 5626), Université de Toulouse, CNRS, UPS, Toulouse, France

<sup>2</sup>Laboratoire de Chimie Quantique, Institut de Chimie, CNRS, Université de Strasbourg, Strasbourg, France

## Faraday Discussions

Cite this: *Faraday Discuss.*, 2020, 224, 402



## PAPER

# Weight dependence of local exchange–correlation functionals in ensemble density-functional theory: double excitations in two-electron systems

Clotilde Marut, <sup>a</sup> Bruno Senjean, <sup>bc</sup> Emmanuel Fromager <sup>d</sup>  
 and Pierre-François Loos <sup>ba</sup>



# Gross-Oliveira-Kohn (GOK) DFT in a three-state ensemble

Ensemble energy:

$$E^{\mathbf{w}} = (1 - w_1 - w_2)E^{(0)} + w_1E^{(1)} + w_2E^{(2)}$$

Excitation energies:

$$\frac{\partial E^{\mathbf{w}}}{\partial w_1} = E^{(1)} - E^{(0)} = \Omega^{(1)} \quad \frac{\partial E^{\mathbf{w}}}{\partial w_2} = E^{(2)} - E^{(0)} = \Omega^{(2)}$$

Ensemble energy in practice:

$$E^{\mathbf{w}} = \min_n \left\{ F^{\mathbf{w}}[n] + \int v_{\text{ext}}(\mathbf{r})n(\mathbf{r})d\mathbf{r} \right\} \quad F^{\mathbf{w}}[n] = T_s^{\mathbf{w}}[n] + E_{\text{Hxc}}^{\mathbf{w}}[n]$$

Derivative discontinuity:

$$\frac{\partial E^{\mathbf{w}}}{\partial w_I} = \mathcal{E}_I^{\mathbf{w}} - \mathcal{E}_0^{\mathbf{w}} + \left. \frac{\partial E_{\text{xc}}^{\mathbf{w}}[n]}{\partial w_I} \right|_{n=n^{\mathbf{w}}(\mathbf{r})} \quad E_{\text{xc}}^{\mathbf{w}}[n] = \int \epsilon_{\text{xc}}^{\mathbf{w}}(n(\mathbf{r}))n(\mathbf{r})d\mathbf{r}$$



# Construction of a weight-dependent LDA functional

Three-state ensemble exchange-correlation functional:

$$\tilde{\epsilon}_{xc}^{w_1, w_2}(n) = (1 - w_1 - w_2)\epsilon_{xc}^{(0)}(n) + w_1\epsilon_{xc}^{(1)}(n) + w_2\epsilon_{xc}^{(2)}(n)$$

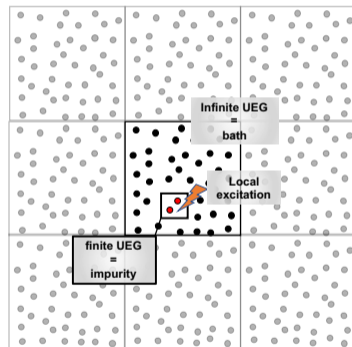
LDA-centered functionals:

$$\bar{\epsilon}_{xc}^{(l)}(n) = \epsilon_{xc}^{(l)}(n) + \epsilon_{xc}^{\text{LDA}}(n) - \epsilon_{xc}^{(0)}(n)$$

$$\tilde{\epsilon}_{xc}^{w_1, w_2}(n) \rightarrow \epsilon_{xc}^{w_1, w_2}(n) = (1 - w_1 - w_2)\bar{\epsilon}_{xc}^{(0)}(n) + w_1\bar{\epsilon}_{xc}^{(1)}(n) + w_2\bar{\epsilon}_{xc}^{(2)}(n)$$

Weight-dependent LDA functional for ensembles “eLDA”:

$$\epsilon_{xc}^{w_1, w_2}(n) = \epsilon_{xc}^{\text{LDA}}(n) + w_1 \left[ \epsilon_{xc}^{(1)}(n) - \epsilon_{xc}^{(0)}(n) \right] + w_2 \left[ \epsilon_{xc}^{(2)}(n) - \epsilon_{xc}^{(0)}(n) \right]$$



## eDFT for charged excitations

PPLB formalism (fractional electrons) [Perdew, Parr, Levy & Balduz PRL 49 (1982) 1691]

$$E^\alpha = (1 - \alpha_1 - \alpha_2)E^N + \alpha_1 E^{N-1} + \alpha_2 E^{N+1}$$

$$n^\alpha = (1 - \alpha_1 - \alpha_2)n^N + \alpha_1 n^{N-1} + \alpha_2 n^{N+1} \Rightarrow \int n^\alpha = N - \alpha_1 + \alpha_2$$

$\Rightarrow$  The exact xc functional does not need to be weight-dependent

$N$ -centered formalism [Senjean & Fromager PRA 98 (2018) 022513]

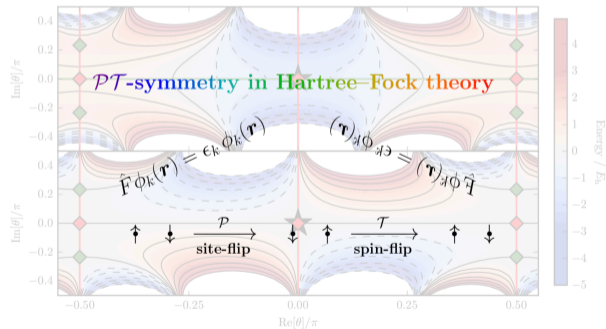
$$E^\xi = \left(1 - \frac{N-1}{N}\zeta_1 - \frac{N+1}{N}\zeta_2\right)E^N + \zeta_1 E^{N-1} + \zeta_2 E^{N+1}$$

$$n^\alpha = \left(1 - \frac{N-1}{N}\zeta_1 - \frac{N+1}{N}\zeta_2\right)n^N + \zeta_1 n^{N-1} + \zeta_2 n^{N+1} \Rightarrow \int n^\xi = N$$

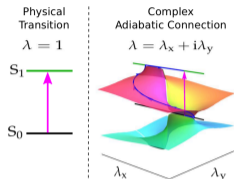
$\Rightarrow$  The exact xc functional must be weight-dependent

## Section 4

## Quantum Chemistry in the Complex Plane



# Quantum Chemistry in the Complex Plane



## Complex adiabatic connection: A hidden non-Hermitian path from ground to excited states

Cite as: *J. Chem. Phys.* **150**, 041103 (2019); doi: [10.1063/1.5085121](https://doi.org/10.1063/1.5085121)  
 Submitted: 9 December 2018 • Accepted: 11 January 2019 •  
 Published Online: 25 January 2019

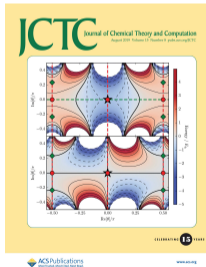


Hugh G. A. Burton,<sup>1,4)</sup> Alex J. W. Thom,<sup>1,5)</sup> and Pierre-François Loos<sup>2,4)</sup>

### AFFILIATIONS

<sup>1</sup>Department of Chemistry, University of Cambridge, Lensfield Road, Cambridge CB2 1EW, United Kingdom

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**JCTC** Journal of Chemical Theory and Computation

Cite This: *J. Chem. Theory Comput.* **2019**, *15*, 4374–4385

[pubs.acs.org/JCTC](https://pubs.acs.org/JCTC)

Article

## Parity-Time Symmetry in Hartree–Fock Theory

Hugh G. A. Burton,<sup>\*,†</sup> Alex J. W. Thom,<sup>‡</sup> and Pierre-François Loos<sup>\*,‡</sup>

<sup>†</sup>Department of Chemistry, University of Cambridge, Lensfield Road, Cambridge CB2 1EW, United Kingdom

<sup>‡</sup>Laboratoire de Chimie et Physique Quantiques (UMR 5626), Université de Toulouse, CNRS, UPS, 31062 Cedex 4 Toulouse, France



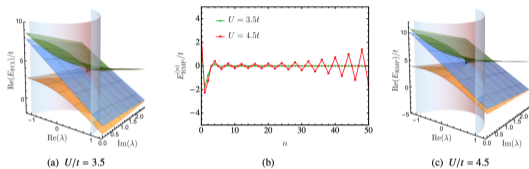
Antoine Marie (M2 ENS)



Hugh Burton (Oxford)

# Perturbation Theory in the Complex Plane [JPCM 33 (2021) 283001]

## Convergence of restricted Møller-Plesset perturbation theory



## Convergence of unrestricted Møller-Plesset perturbation theory

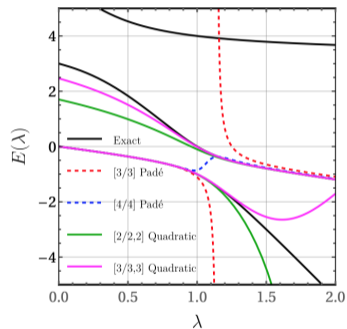
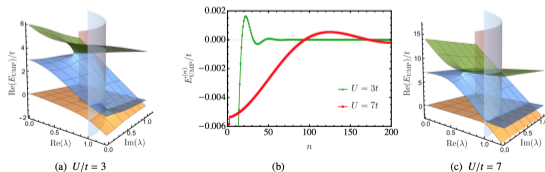
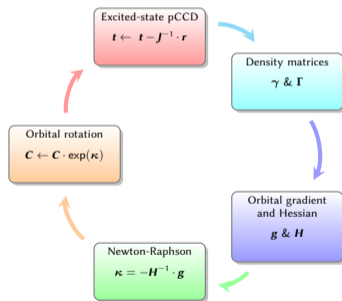


FIG. 12: UMP energies in the Hubbard dimer as a function of  $\lambda$  obtained using various approximants at  $U/t = 3$ .

## Section 5

## Coupled-cluster theory for excited states



# Coupled-cluster theory for excited states



Fábris Kossoski  
(Postdoc ERC)



Antoine Marie  
(M2 ENS)

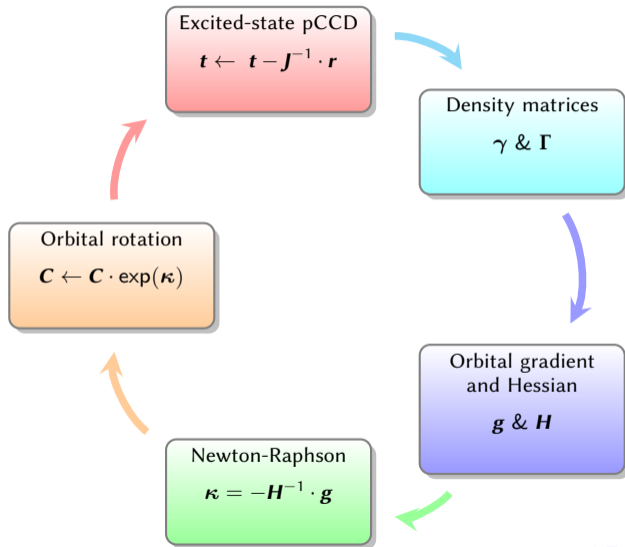


Anthony Scemama  
(LCPQ)

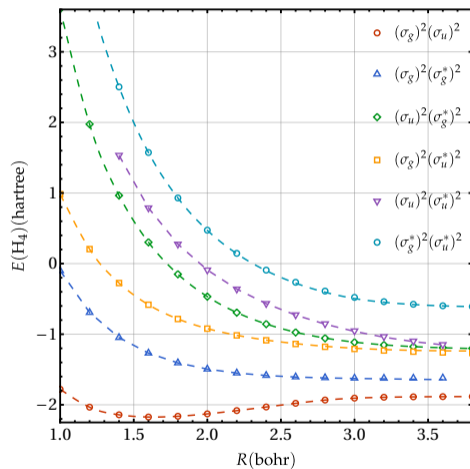


Michel Caffarel  
(LCPQ)

## State-specific orbital-optimized TpCCD (oo-TpCCD) for excited states





Stretching linear H<sub>4</sub>/STO-6G: TpCCD vs DOCI with state-specific TpCCD reference

Lowest doubly-excited state of CH<sup>+</sup>

molecule	method	$\Delta E$ (eV)	$\Delta\Delta E$ (eV)
CH <sup>+1</sup>	$\Delta oo$ -TpCCD	8.36	-0.19
	FCI <sup>2</sup>	8.55	0
	EOM-CCSDT <sup>3</sup>	8.62	+0.07
	EOM-CCSDt <sup>3</sup>	8.64	+0.09
	EOM-oo-pCCD-LCCSD <sup>4</sup>	8.84	+0.29
	EOM-pCCD-LCCSD <sup>4</sup>	7.61	-0.94
	CC3 <sup>5</sup>	8.78	+0.23

<sup>1</sup>Basis set and geometry taken from Olsen et al. CPL 154 (1989) 380

<sup>2</sup>Results from Olsen et al. CPL 154 (1989) 380

<sup>3</sup>Results from Kowalski & Piecuch, CPL 347(2001) 237

<sup>4</sup>Results from Boguslawski, JCTC 15 (2019) 18

<sup>5</sup>Results from Christiansen et al. JCP 103 (1995) 7429

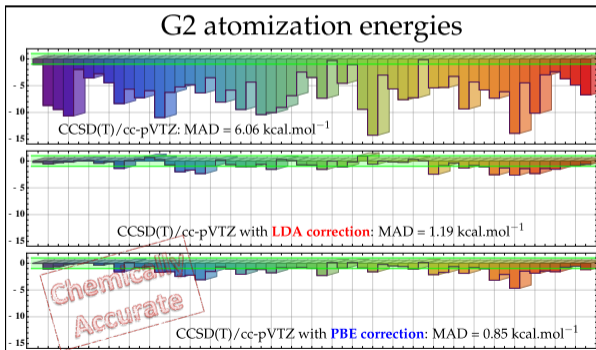
## More doubly-excited states for molecules (6-31+G\*)...

molecule	method	$\Delta E$ (eV)	$\Delta\Delta E$ (eV)
BH	$\Delta_{oo}$ -TpCCD	7.35	+0.24
	FCI	7.11	0
	EOM-CCSDTQ	7.12	+0.01
	EOM-CCSDT	7.15	+0.04
	CC3	7.30	+0.19
HNO	$\Delta_{oo}$ -TpCCD	4.49	-0.02
	FCI <sup>1</sup>	4.51	0
	EOM-CCSDTQ <sup>1</sup>	4.54	+0.03
	EOM-CCSDT <sup>1</sup>	4.81	+0.30
	CC3 <sup>1</sup>	5.28	+0.77
H <sub>3</sub> C-NO	$\Delta_{oo}$ -TpCCD	4.66	-0.20
	FCI <sup>1</sup>	4.86	0
	EOM-CCSDT <sup>1</sup>	5.26	+0.40
	CC3 <sup>1</sup>	5.73	+0.87
H <sub>2</sub> C=O	$\Delta_{oo}$ -TpCCD	11.26	+0.40
	FCI <sup>1</sup>	10.86	0
	EOM-CCSDTQ <sup>1</sup>	10.87	+0.01
	EOM-CCSDT <sup>1</sup>	11.10	+0.24
	CC3 <sup>1</sup>	11.49	+0.63

<sup>1</sup>Results and geometries from Loos et al. JCTC 15 (2019) 1939

## Section 6

## Basis set incompleteness correction



## Ground-state properties

## A Density-Based Basis-Set Correction for Wave Function Theory

Pierre-François Loos,<sup>\*,†</sup> Barthélémy Pradines,<sup>‡,§</sup> Anthony Scemama,<sup>†</sup> Julien Toulouse,<sup>\*,‡</sup>  
and Emmanuel Giner<sup>\*,‡</sup>

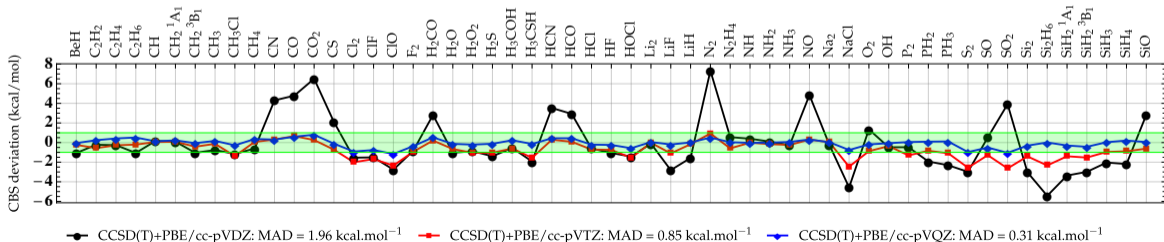
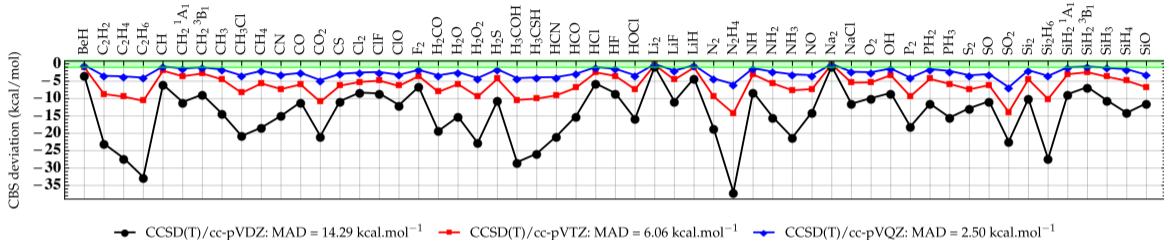
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# Ground-state properties



## Neutral excitations

# Chemically accurate excitation energies with small basis sets

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## AFFILIATIONS

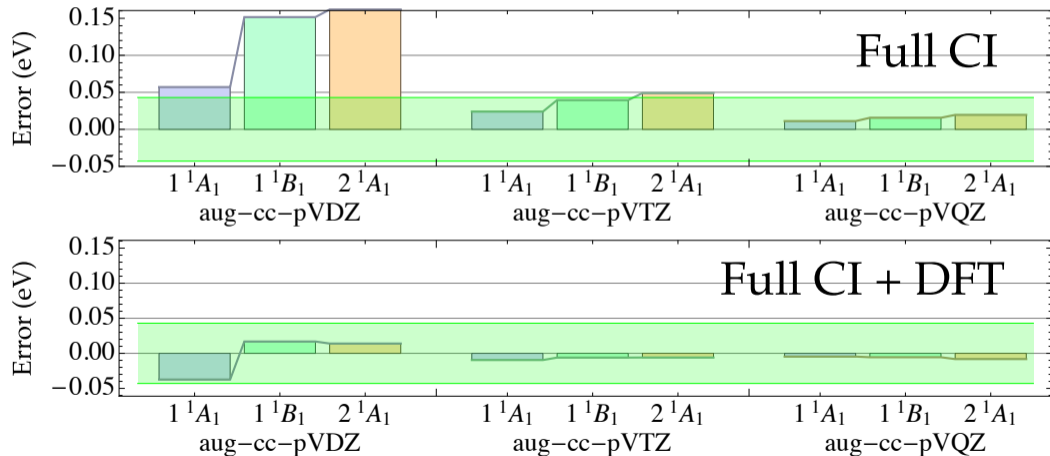
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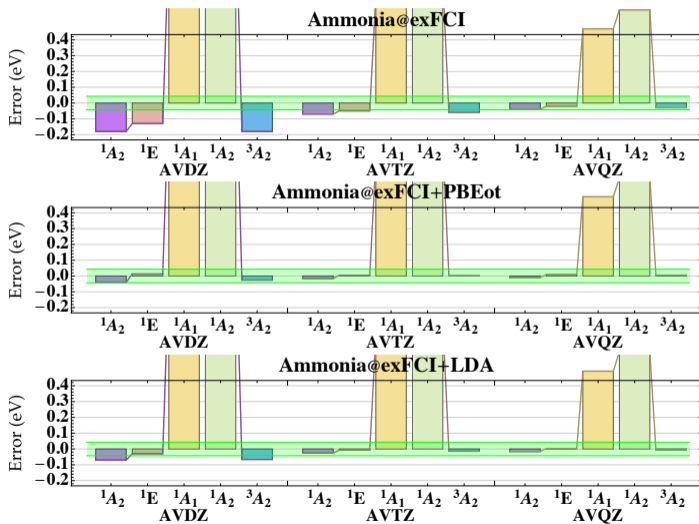
## Neutral excitations

## Adiabatic energies of methylene





## Neutral excitations



## Charged excitations



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Article

# Density-Based Basis-Set Incompleteness Correction for GW Methods

Pierre-François Loos,\* Barthélemy Pradines, Anthony Scemama, Emmanuel Giner, and Julien Toulouse\*



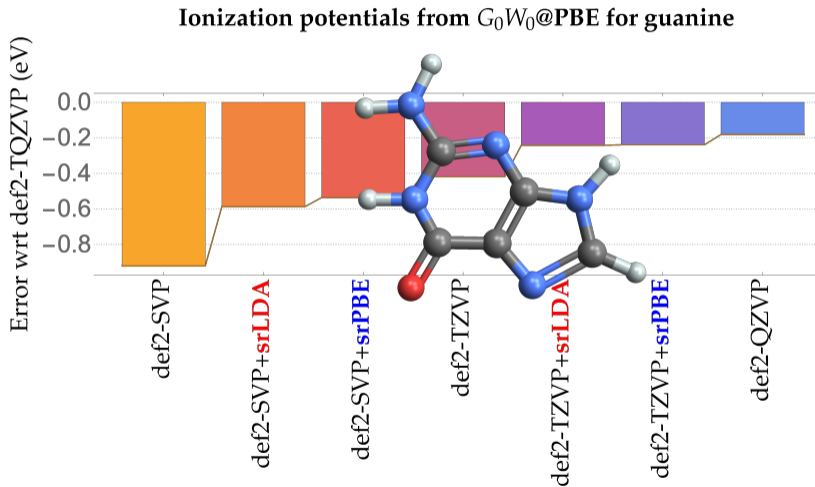
Cite This: *J. Chem. Theory Comput.* 2020, 16, 1018–1028



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## Charged excitations



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