# A Mountaineering Strategy to Excited States: Revising Reference Values with EOM-CC4 

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#### Abstract

In the framework of the computational determination of highly accurate vertical excitation energies in small organic compounds, we explore the possibilities offered by the equation-of-motion formalism relying on the approximate fourth-order coupled-cluster (CC) method, CC4. We demonstrate, using an extended set of more than 200 reference values based on CC including up to quadruples excitations (CCSDTQ), that CC4 is an excellent approximation to CCSDTQ for excited states with a dominant contribution from single excitations with an average deviation as small as 0.003 eV . We next assess the accuracy of several additive basis set correction schemes, in which vertical excitation energies obtained with a compact basis set and a high-order CC method are corrected with lowerorder CC calculations performed in a larger basis set. Such strategies are  found to be overall very beneficial, though their accuracy depends significantly on the actual scheme. Finally, CC4 is employed to improve several theoretical best estimates of the QUEST database for molecules containing between four and six (nonhydrogen) atoms, for which previous estimates were computed at the CCSDT level.


## 1. INTRODUCTION

Defining sets of high-quality reference values that can be employed to reliably assess the pros and cons of lower-cost theoretical methods is a very popular and useful research line in quantum chemistry. ${ }^{1-11}$ Although experimental values may constitute natural references for some properties (e.g., thermodynamical and kinetic data), ${ }^{12-16}$ it is often welcome to rely on state-of-the-art electronic structure methods to produce reference values for other properties. Of course, this latter approach is intrinsically limited by the computational cost of these high-accuracy models. However, it has the undeniable advantage to allow well-grounded comparisons within a unique, well-defined set of parameters. ${ }^{17-20}$ Indeed, one can perform comparisons with exactly the same geometries, basis set, solvent model, etc. Purely theoretical reference values are especially useful for electronic excited states (ESs), as the most straightforwardly accessible theoretical values, namely, vertical transition energies (VTEs), are not directly measurable experimentally. ${ }^{21}$ This explains why the determination of accurate VTEs has been an active and productive avenue of research during the past three decades, with, in particular, valuable works from Roos ${ }^{22-24}$ and Thiel's ${ }^{25-27}$ groups.
Since 2018, our groups have made several contributions in this field, ${ }^{11,28-32}$ and it eventually led to the creation of the QUEST database (see https://lcpq.github.io/QUESTDB_ website) that contains a large panel of reference VTEs for molecules containing from 1 to 10 nonhydrogen atoms. ${ }^{33} \mathrm{At}$
the present stage, the QUEST database includes more than 500 theoretical best estimates (TBEs) for diverse ESs (singlet, doublet, and triplet; valence and Rydberg; charge transfer, singly- and doubly excited states) that have been established with the aug-cc-pVTZ basis set. Typically, the TBEs contained in the QUEST database are produced using VTEs computed with the selected configuration interaction algorithm named "Configuration Interaction using a Perturbative Selection made Iteratively" (CIPSI) ${ }^{34-39}$ to obtain near full configuration interaction (FCI) quality VTEs for systems containing from 1 to 3 nonhydrogen atoms, coupled cluster (CC) with singles, doubles, triples and, quadruples (CCSDTQ) ${ }^{40}$ for molecules encompassing 4 nonhydrogen atoms, and CC with singles, doubles, and triples (CCSDT) ${ }^{41-45}$ for larger derivatives. Popular basis set correction schemes have often been applied. For example, the CCSDTQ VTEs computed with a double- $\zeta$ basis set were corrected thanks to CCSDT values obtained with a triple- $\zeta$ basis set. Most TBEs included in the QUEST database were estimated to be chemically accurate (corresponding to $1 \mathrm{kcal} \cdot \mathrm{mol}^{-1}$ or 0.04 eV error), with a typical error

[^0]
bar of $\pm 0.03 \mathrm{eV}$. The reference values included in QUEST have been used by various groups, for example, to (i) assess the relative accuracies of third-order, ${ }^{46}$ multireference, ${ }^{47}$ and other emerging ${ }^{48-53}$ methods, (ii) quantify the accuracy of local hybrids for triplet ESs, ${ }^{54}$ (iii) determine the relative performance of several hybrid ${ }^{55}$ and double hybrid ${ }^{56-59}$ functionals, and (iv) evaluate the potential of orbital-optimized density functional theory for double excitations. ${ }^{60,61}$
Despite burning an unreasonable number of CPU hours during the past 4 years, we could hardly "do better" than what is described above, as we rapidly hit the computational wall of both high-order CC schemes and/or large CIPSI calculations. For instance, CCSDTQ formally scales as $O\left(N^{10}\right)$ (where $N$ is the number of basis functions), and determining VTEs in molecules like furan or thiophene is nearly impossible even in a double- $\zeta$ basis set. In an effort to go one step further, we explored very recently, and for the first time, the performance of the approximate fourth-order CC model, CC4, ${ }^{62}$ in the context of ESs. ${ }^{63}$ From a theoretical point of view, CC4 can be viewed as an approximation of CCSDTQ that still includes iterative quadruples, but neglects the calculation of the most expensive components (and avoids the storage of the higherexcitation amplitudes), allowing to reduce the formal scaling by 1 order of magnitude to $O\left(N^{9}\right) .{ }^{62}$ In our first investigation, ${ }^{63}$ we considered very small systems $\left(\mathrm{BH}, \mathrm{BF}, \mathrm{CO}, \mathrm{HCl}, \mathrm{H}_{2} \mathrm{O}\right.$, $\mathrm{H}_{2} \mathrm{~S}, \mathrm{~N}_{2}$, and $\mathrm{NH}_{3}$ ) for which well-converged CIPSI calculations were achievable for 25 ESs. This preliminary study indicates that CC4 indeed provides highly competitive VTEs, as it allows to significantly reduce the CCSDT error as compared to FCI, with final deviations-for ESs with a dominant single excitation character-only slightly larger than their CCSDTQ counterparts.

In the present contribution, which is, to our knowledge, only the second work presenting CC4 calculations for ESs, we build on our previous study, ${ }^{63}$ and we aim at (i) evaluating the performance of CC4 for a more significant set of molecules and a larger variety of ESs, (ii) assessing the accuracy of additive basis set correction procedures based on CC4, and (iii) providing improved TBEs for many ESs included in the QUEST database.

## 2. COMPUTATIONAL DETAILS

All calculations presented here rely on the frozen-core approximation and the high-quality ground-state geometries extracted from the QUEST database. ${ }^{33}$ For the sake of reproducibility, Cartesian coordinates for all systems displayed in Figure 1 are reproduced in the Supporting Information (SI). Note that, below, we do not specify the equation-of-motion (EOM) prefix for the CC calculations, although all ES calculations are performed using this formalism.

Most of our CC calculations have been performed with CFOUR, ${ }^{64,65}$ which provides an efficient implementation of high-order CC methods up to quadruples. ${ }^{66,67}$ For all considered systems, we performed, when technically achievable, CC2,,$^{68,69}$ CCSD ${ }^{70-74} \operatorname{CC} 3,{ }^{75-77}$ CCSDT, ${ }^{41-45}$ CC4, ${ }^{62,78}$ and CCSDTQ ${ }^{40,78}$ calculations using three Gaussian basis sets: $6-31+G(d)$, aug-cc-pVDZ, and aug-cc-pVTZ. For the sake of conciseness, these basis sets are sometimes dubbed as "Pop", "AVDZ", and "AVTZ" in the following. These three bases were selected because they contain both polarization and diffuse and are widely available, aug-cc-pVTZ being typically


Figure 1. Representation of the systems investigated in the present study.
viewed as providing results close from the basis set limit for VTEs, except for very diffuse Rydberg states.

The identification of all states follows the QUEST database. ${ }^{33}$ The separation between valence and Rydberg transitions was performed by examining the dominant unoccupied orbital contribution in the CCSD and CC3 results, which typically gives a clear trend, except for a few high-lying ESs for which mixing can appear. The doubleexcitation character of all transitions was probed by using $\% T_{1}$ the percentage of single excitations as given by CC3. Note that genuine or pure double ESs have a negligible $\% T_{1}$ and are unseen by CC2 and CCSD approaches.

In the statistical analysis presented below, we report the usual indicators: the mean signed error (MSE), the mean absolute error (MAE), as well as largest positive and negative deviations [ $\operatorname{Max}(+)$ and $\operatorname{Max}(-)$, respectively].

CCSDTQP/6-31+G(d) calculations have been performed with MRCC $^{79,80}$ for the molecules encompassing two nonhydrogen atoms (see Figure 1). These values are reported in Table S1 of the SI. Considering all 32 VTEs and taking these CCSDTQP values as references, we obtain a MAE of 0.003 eV for the CCSDTQ data determined with the same basis set. Unsurprisingly, the largest error, 0.023 eV , comes from the pure $(n, n) \rightarrow\left(\pi^{*}, \pi^{*}\right)$ doubly excited state of

Table 1. Statistical Analysis Obtained Using Various Correction Schemes ${ }^{a}$

| approximating | using | and correcting with | MSE | MAE | Max(+) | Max(-) | useful? |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CCSDTQ/aug-cc-pVTZ | CC4/aug-cc-pVTZ | no correction | 0.002 | 0.003 | 0.012 | -0.003 |  |
|  |  | + [CCSDTQ - CC4]/aug-cc-pVDZ | 0.000 | 0.001 | 0.003 | -0.003 | 22/31 |
|  |  | $+[\mathrm{CCSDTQ}-\mathrm{CC} 4] / 6-31+\mathrm{G}(\mathrm{d})$ | 0.001 | 0.002 | 0.006 | -0.004 | 19/31 |
|  | CCSDT/aug-cc-pVTZ | no correction | -0.001 | 0.017 | 0.107 | -0.049 |  |
|  |  | + [CCSDTQ - CCCST]/aug-cc-pVDZ | 0.003 | 0.004 | 0.022 | -0.004 | 27/31 |
|  |  | + [CCSDTQ - CCSDT]/6-31+G(d) | 0.001 | 0.003 | 0.030 | -0.005 | 25/31 |
|  | CC3/aug-cc-pVTZ | no correction | 0.006 | 0.017 | 0.124 | -0.034 |  |
|  |  | $+[\mathrm{CCSDTQ}-\mathrm{CC} 3] /$ aug-cc-pVDZ | 0.002 | 0.004 | 0.019 | -0.003 | 28/31 |
|  |  | $+[\mathrm{CCSDTQ}-\mathrm{CC} 3] / 6-31+\mathrm{G}(\mathrm{d})$ | 0.001 | 0.004 | 0.020 | -0.008 | 26/31 |
|  | CCSD/aug-cc-pVTZ | no correction | 0.071 | 0.071 | 0.273 | -0.005 |  |
|  |  | + [CCSDTQ - CCSD $] /$ aug-cc-pVDZ | 0.022 | 0.024 | 0.066 | -0.011 | 26/31 |
|  |  | + [CCSDTQ - CCSD $] / 6-31+\mathrm{G}(\mathrm{d})$ | 0.010 | 0.032 | 0.061 | -0.042 | 25/31 |
|  | CC2/aug-cc-pVTZ | no correction | -0.008 | 0.215 | 0.348 | -0.695 |  |
|  |  | + [CCSDTQ - CC2]/aug-cc-pVDZ | 0.032 | 0.032 | 0.196 | -0.001 | 29/31 |
|  |  | $+[\mathrm{CCSDTQ}-\mathrm{CC} 2] / 6-31+\mathrm{G}(\mathrm{d})$ | -0.005 | 0.051 | 0.108 | -0.353 | 29/31 |
| CC4/aug-cc-pVTZ | CCSDT/aug-cc-pVTZ | no correction | 0.004 | 0.016 | 0.095 | -0.055 |  |
|  |  | $+[\mathrm{CC} 4-\mathrm{CCSDT}] /$ aug-cc-pVDZ | 0.004 | 0.004 | 0.022 | -0.005 | 52/59 |
|  |  | $+[\mathrm{CC} 4-\mathrm{CCSDT}] / 6-31+\mathrm{G}(\mathrm{d})$ | 0.002 | 0.004 | 0.028 | -0.009 | 55/59 |
|  | CC3/aug-cc-pVTZ | no correction | 0.006 | 0.016 | 0.112 | -0.043 |  |
|  |  | $+[\mathrm{CC} 4-\mathrm{CC} 3] /$ aug-cc-pVDZ | 0.002 | 0.003 | 0.019 | -0.004 | 52/59 |
|  |  | $+[\mathrm{CC} 4-\mathrm{CC} 3] / 6-31+\mathrm{G}(\mathrm{d})$ | 0.000 | 0.004 | 0.018 | -0.010 | 52/59 |
|  | CCSD/aug-cc-pVTZ | no correction | 0.083 | 0.084 | 0.261 | -0.009 |  |
|  |  | + [CC4 - CCSD $] /$ aug-cc-pVDZ | 0.018 | 0.019 | 0.063 | -0.009 | 54/59 |
|  |  | $+[\mathrm{CC} 4-\mathrm{CCSD}] / 6-31+\mathrm{G}(\mathrm{d})$ | 0.008 | 0.023 | 0.081 | -0.041 | 51/59 |
|  | CC2/aug-cc-pVTZ | no correction | 0.053 | 0.193 | 0.427 | -0.701 |  |
|  |  | $+[\mathrm{CC} 4-\mathrm{CC} 2] /$ aug-cc-pVDZ | 0.027 | 0.029 | 0.196 | -0.016 | 56/59 |
|  |  | $+[\mathrm{CC} 4-\mathrm{CC} 2] / 6-31+\mathrm{G}(\mathrm{d})$ | 0.000 | 0.049 | 0.183 | -0.352 | 54/59 |
| CCSDT/aug-cc-pVTZ | CC3/aug-cc-pVTZ | no correction | 0.004 | 0.015 | 0.073 | -0.046 |  |
|  |  | $+[\operatorname{CCSDT}-\mathrm{CC} 3] / a u g-c c-p V D Z$ | -0.002 | 0.003 | 0.007 | -0.016 | 109/118 |
|  |  | $+[\operatorname{CCSDT}-\mathrm{CC} 3] / 6-31+\mathrm{G}(\mathrm{d})$ | -0.002 | 0.005 | 0.011 | -0.036 | 102/118 |
|  | CCSD/aug-cc-pVTZ | no correction | 0.114 | 0.114 | 0.539 | 0.010 |  |
|  |  | $+[\operatorname{CCSDT}-\mathrm{CCSD}] /$ aug-cc-pVDZ | 0.022 | 0.025 | 0.073 | -0.015 | 115/118 |
|  |  | $+[\mathrm{CCSDT}-\mathrm{CCSD}] / 6-31+\mathrm{G}(\mathrm{d})$ | 0.012 | 0.027 | 0.071 | -0.044 | 112/118 |
|  | CC2/aug-cc-pVTZ | no correction | -0.007 | 0.191 | 0.495 | -0.675 |  |
|  |  | + [CCSDT - CC2]/aug-cc-pVDZ | 0.020 | 0.023 | 0.174 | -0.020 | 115/118 |
|  |  | $+[\mathrm{CCSDT}-\mathrm{CC} 2] / 6-31+\mathrm{G}(\mathrm{d})$ | -0.019 | 0.047 | 0.167 | -0.355 | 106/118 |

${ }^{a}$ MSE, MAE, and maximal deviations (in eV ) are obtained with respect to the reference method given in the leftmost column. The number of reference ESs considered at the CCSDTQ/aug-cc-pVTZ, CC4/aug-cc-pVTZ, and CCSDT/aug-cc-pVTZ levels is 31 , 59 , and 118 , respectively. In the rightmost column, we provide the number of states for which the correction provides smaller (or equal) absolute errors as compared to the noncorrected scheme (rows labeled as "no correction").
nitroxyl for which the CC expansion obviously converges slower. Removing this pathological case yields a MAE of 0.002 eV with all errors below 0.010 eV , clearly confirming the quality of the CCSDTQ estimates.

## 3. METHODOLOGICAL ASPECTS

3.1. How Does CC4 Compare to CCSDTQ? Given that CC4 is an approximation of CCSDTQ and that the accuracy of the latter is recognized as exceptional for ESs with a dominant contribution from singly excited determinants (see above), it seems natural to assess the performance of CC4 with respect to CCSDTQ. Considering the values listed in the SI (with the three basis sets and all compounds), we have 220 CCSDTQ reference values at hand. Using these reference values, one obtains a MSE of 0.002 eV and a MAE as small as 0.003 eV for CC4. As one could have foreseen, ${ }^{63}$ the largest deviations between CC4 and CCSDTQ originate from the genuine double excitations in nitroxyl, nitrosomethane, and glyoxal for which CC-based methods would likely not be chosen in
practical applications. Removing these three states, but keeping all other cases including the $2^{1} A_{g}$ dark state of butadiene, leads to a MSE of 0.001 eV and a MAE of 0.003 eV for CC4. Interestingly, in our earlier work devoted to very small compounds, we obtained the same MAE of 0.003 eV for CC4 (against CCSDTQP). ${ }^{63}$ Clearly, one can state that CC4 is a very good approximation of CCSDTQ for the ESs that are not dominated by doubly excited determinants.

### 3.2. Is CC4 Significantly More Accurate Than CCSDT

 and CC3? The next natural question is to determine if CC4 is worth its cost. Indeed, CC4 formally scales as $O\left(N^{9}\right)$ which is more costly than both CCSDT and CC3 [which scale as $O\left(N^{8}\right)$ and $O\left(N^{7}\right)$, respectively]. Using again the available CCSDTQ values as references, we obtain MSE and MAE of 0.000 and 0.015 eV for CCSDT and 0.006 and 0.018 eV for CC3, again discarding from the set the true double excitations of nitroxyl, nitrosomethane, and glyoxal. It is no surprise that, on the one hand, both CC3 and CCSDT deliver results that would be rated as accurate enough for many applications, and,on the other hand, CC3 is an excellent approximation of CCSDT. ${ }^{33,81,82}$ The most valuable observation for our purposes is that CC4 indeed lowers the CCSDT and CC3 deviations by a factor of 5 , which we consider a very significant improvement when one aims at producing highly accurate reference values.
3.3. Are CC4 Basis Set Effects Transferable? When defining values for benchmarking purposes, it is obvious from the discussion above that one can safely employ CC4/aug-ccpVTZ as reference when neither well-converged CIPSI/aug-ccpVTZ nor CCSDTQ/aug-cc-pVTZ calculations are technically achievable. As detailed below, this is the case, for example, for some of the molecules encompassing four nonhydrogen atoms. However, as customary in the field, ${ }^{31,32,56,78,83-89}$ one can employ a double- $\zeta$ VTE computed with a CC model including quadruples in order to correct a triple- $\zeta$ VTE obtained with a quadruple-free CC method.
To test such strategies, we report in Table 1 the statistical analysis obtained with various correction schemes. We consider as reference the actual results obtained with the target method (leftmost column of Table 1). These reference values are available in Tables S3, S6, and S9 of the SI. In the rightmost column, we provide the number of states for which the correction yields smaller (or equal) absolute errors as compared to the noncorrected scheme (rows labeled as "no correction").

Globally, the data listed in Table 1 are quite appealing and one notices three general trends. First, adding corrections for higher-order excitations systematically decrease the MAE and improves the (vast) majority of the estimates. For example, while CC2/aug-cc-pVTZ returns a MAE of 0.215 eV as compared to CCSDTQ/aug-cc-pVTZ, correcting the former values with the differences between the VTEs computed with these two models but with the much smaller $6-31+G(d)$ basis set allows reducing the error to 0.051 eV and improves 29 of the 31 VTEs. Second, the more refined the starting point, the more accurate the final estimate. For example, using the same approach as above but starting from CC3/aug-cc-pVTZ rather than CC2/aug-cc-pVTZ would cut down the MAE from 0.051 to 0.004 eV . Third, performing the correction with aug-ccpVDZ typically yields smaller statistical deviations than with 6$31+G(d)$, though the difference between the two approaches is small.

Given that CC4 is such a stunning approximation of CCSDTQ for ESs with a single excitation character, it is particularly relevant to investigate CC4/aug-cc-pVTZ estimates based on CCSDT and CC3, i.e.:

$$
\begin{align*}
& \Delta \tilde{E}_{\mathrm{AVTZ}}^{\mathrm{CC} 4} \simeq \Delta E_{\mathrm{AVTZ}}^{\mathrm{CCSDT}}+\left[\Delta E_{\mathrm{AVDZ}}^{\mathrm{CC4}}-\Delta E_{\mathrm{AVDZ}}^{\mathrm{CCSDT}}\right]  \tag{1a}\\
& \Delta \tilde{E}_{\mathrm{AVTZ}}^{\mathrm{CC4}} \simeq \Delta E_{\mathrm{AVTZ}}^{\mathrm{CCSDT}}+\left[\Delta E_{\mathrm{Pop}}^{\mathrm{CC} 4}-\Delta E_{\mathrm{Pop}}^{\mathrm{CCDT}}\right]  \tag{1b}\\
& \Delta \tilde{E}_{\mathrm{AVTZ}}^{\mathrm{CC} 4} \simeq \Delta E_{\mathrm{AVTZ}}^{\mathrm{CC} 3}+\left[\Delta E_{\mathrm{AVDZ}}^{\mathrm{CC} 4}-\Delta E_{\mathrm{AVDZ}}^{\mathrm{CC} 3}\right]  \tag{1c}\\
& \Delta \tilde{E}_{\mathrm{AVTZ}}^{\mathrm{CC} 4} \simeq \Delta E_{\mathrm{AVTZ}}^{\mathrm{CC} 3}+\left[\Delta E_{\mathrm{Pop}}^{\mathrm{CC4} 4}-\Delta E_{\mathrm{Pop}}^{\mathrm{CC} 3}\right] \tag{1d}
\end{align*}
$$

Looking at Table 1, one notices that all four approaches deliver very comparable error patterns, with MAE not larger than 0.004 eV , and global improvement over the uncorrected CCSDT/aug-cc-pVTZ and CC3/aug-cc-pVTZ data, the errors being cut down by a factor of 3 . Of course, as can be deduced from the last column, not all values are more accurate when
adding corrections obtained with smaller basis sets, but improvements are observed for the vast majority of the cases.

Additionally, although this is not the focus of the present work, we stress that the following expressions:

$$
\begin{align*}
& \Delta \tilde{E}_{\mathrm{AVTZ}}^{\mathrm{CCSDT}} \simeq \Delta E_{\mathrm{AVTZ}}^{\mathrm{CC3}}+\left[\Delta E_{\mathrm{AVDZ}}^{\mathrm{CCSDT}}-\Delta E_{\mathrm{AVDZ}}^{\mathrm{CC3}}\right]  \tag{2a}\\
& \Delta \tilde{E}_{\mathrm{AVTZ}}^{\mathrm{CCSDT}} \simeq \Delta E_{\mathrm{AVTZ}}^{\mathrm{CC3}}+\left[\Delta E_{\mathrm{Pop}}^{\mathrm{CCSDT}}-\Delta E_{\mathrm{Pop}}^{\mathrm{CC} 3}\right]
\end{align*}
$$

are extremely effective with MAE of 0.003 and 0.005 eV and small maximal deviations as compared to the true CCSDT/ aug-cc-pVTZ transition energies. The fact that these two approximations are very accurate is certainly valuable, as both $\mathrm{CC} 3 /$ aug-cc-pVTZ and CCSDT/6-31+G(d) calculations are technically achievable on systems with up to approximately 10-12 nonhydrogen atoms, allowing to significantly expand the number of TBEs based on CCSDT/aug-cc-pVTZ that are included in the QUEST database for larger systems.

The take-home message of this section is that, while these correction schemes (which are widespread in the CC community) are generally valuable to produce accurate VTEs, their overall accuracy depends ultimately on the quality of the starting point.

## 4. REFINED THEORETICAL BEST ESTIMATES

Given the above observations, it is worth reexamining our previous TBEs with the additional accuracy provided by CC4.
4.1. Linear Systems. The QUEST database includes three linear systems encompassing four nonhydrogen atoms: cyanoacetylene, cyanogen, and diacetylene. ${ }^{11,33}$ For these three systems CC4/aug-cc-pVTZ calculations could be performed, whereas CCSDTQ/aug-cc-pVTZ calculations remain beyond reach (see Table S9 in the SI for raw data). We have, therefore, computed new TBEs on the basis of these CC4 data, applying the very trustworthy basis set correction (see top lines of Table 1).

$$
\begin{equation*}
\Delta E_{\mathrm{AVTZ}}^{\mathrm{TBE}}=\Delta E_{\mathrm{AVTZ}}^{\mathrm{CC} 4}+\left[\Delta E_{\mathrm{AVDZ}}^{\mathrm{CCSDTQ}}-\Delta E_{\mathrm{AVDZ}}^{\mathrm{CC} 4}\right] \tag{3}
\end{equation*}
$$

The results for seven ESs of these three linear molecules are listed in Table 2. It can be seen that the variations are negligible, with three TBEs unchanged and four decreasing by 0.01 eV .
4.2. Four Nonhydrogen Compounds. For most nonlinear compounds encompassing four nonhydrogen nuclei, the TBEs of the QUEST database were obtained with CCSDT/

Table 2. TBE/aug-cc-pVTZ (in eV) Established for the Linear Systems Encompassing Four Nonhydrogen Atoms ${ }^{a}$

|  |  | TBEs |  |  |  |
| :--- | :--- | :---: | :---: | ---: | ---: |
| molecule | transition | nature | new | prev. | diff. |
| cyanoacetylene | ${ }^{1} \Sigma^{-}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 5.79 | 5.80 | -0.01 |
|  | ${ }^{1} \Delta\left(\pi \rightarrow \pi^{*}\right)$ | Val | 6.07 | 6.07 | 0.00 |
| cyanogen | ${ }^{1} \Sigma_{u}^{-}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 6.38 | 6.39 | -0.01 |
|  | ${ }^{1} \Delta_{u}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 6.65 | 6.66 | -0.01 |
|  | ${ }^{1} \Sigma_{u}^{-}\left(\pi \rightarrow \pi^{*}\right)[\mathrm{F}]^{b}$ | Val | 5.04 | 5.05 | -0.01 |
| diacetylene | ${ }^{1} \Sigma_{u}^{-}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 5.33 | 5.33 | 0.00 |
|  | ${ }^{1} \Delta_{u}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 5.61 | 5.61 | 0.00 |

a"Val" stands for valence ESs. TBEs obtained with eq 3 using data from the SI. The previous TBEs taken from ref 11 are given for comparison. ${ }^{b}$ Fluorescence from the optimized excited-state geometry.
aug-cc-pVTZ corrected using CCSDTQ/6-31+G(d) to take into account the quadruples contribution. ${ }^{11,33}$ Given the results gathered in Table 1, it is not clear that applying CC4/aug-cc-pVDZ corrections would improve significantly these original estimates. However, for a few compounds, the QUEST TBEs do not include corrections from the quadruples, as they were computed at the CCSDT/aug-cc-pVTZ level "only". Hence, we have considered these derivatives and employed eq 1a to determine new TBEs. The results are listed in Table 3 and compared to the previous estimates.

Table 3. TBE/aug-cc-pVTZ (in eV) Established for Nonlinear Systems Containing Four Nonhydrogen Atoms Obtained Using Eq $1 a^{a}$

|  |  | TBEs |  |  |  |
| :--- | :---: | :--- | :--- | :--- | ---: |
| molecule | transition | nature | new | prev. | diff. |
| acrolein | ${ }^{1} A^{\prime \prime}\left(n \rightarrow \pi^{*}\right)$ | Val | 3.72 | $3.78^{b}$ | -0.06 |
|  | ${ }^{1} A^{\prime}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 6.67 | 6.69 | -0.02 |
|  | ${ }^{1} A^{\prime \prime}\left(n \rightarrow \pi^{*}\right)$ | Val | 6.69 | $6.72^{c}$ | -0.03 |
|  | ${ }^{1} A^{\prime}(n \rightarrow 3 s)$ | Ryd | 7.11 | 7.08 | +0.03 |
|  | ${ }^{1} A^{\prime}\left(\pi \rightarrow \pi^{*}\right)$ | $\mathrm{Val}^{d}$ | 7.93 | $7.87^{b}$ | +0.06 |
| carbonylfluoride | ${ }^{1} A_{2}\left(n \rightarrow \pi^{*}\right)$ | Val | 7.29 | $7.31^{e}$ | -0.02 |
| cyanoformaldehyde | ${ }^{1} A^{\prime \prime}\left(n \rightarrow \pi^{*}\right)$ | Val | 3.82 | 3.81 | +0.01 |
|  | ${ }^{1} A^{\prime \prime}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 6.43 | 6.46 | -0.03 |
| isobutene | ${ }^{1} B_{1}(\pi \rightarrow 3 s)$ | Ryd | 6.48 | 6.46 | +0.02 |
|  | ${ }^{1} A_{1}(\pi \rightarrow 3 p)$ | Ryd | 7.01 | 7.01 | 0.00 |
| propynal | ${ }^{1} A^{\prime \prime}\left(n \rightarrow \pi^{*}\right)$ | Val | 3.81 | 3.80 | +0.01 |
|  | ${ }^{1} A^{\prime \prime}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 5.51 | 5.54 | -0.03 |
| thioacrolein | ${ }^{1} A^{\prime \prime}\left(n \rightarrow \pi^{*}\right)$ | Val | 2.10 | 2.11 | -0.01 |
| thiopropynal | ${ }^{1} A^{\prime \prime}\left(n \rightarrow \pi^{*}\right)$ | Val | 2.02 | 2.03 | -0.01 |

a"Val" and "Ryd" stand for valence and Rydberg ESs. Raw data are given in the SI. The previous TBE/aug-cc-pVTZ values taken from refs $11,29,30$, and 33 are given for comparison. ${ }^{b}$ TBEs obtained from a basis set corrected CIPSI/6-31+G(d) estimate. The CCSDT/aug-cc-pVTZ values for these two specific states are 3.73 and 8.01 eV . ${ }^{c}$ Considered as unsafe in ref $11 .{ }^{d}$ ES with a significant double excitation character. ${ }^{e}$ TBE obtained from a basis set corrected CIPSI/ $6-31+G(d)$ estimate.

Although the differences listed in the rightmost column of Table 3 have different signs, the impact of CC4 is typically a small decrease of the previous estimates, hinting that CCSDT tends to overestimate the VTEs. This is consistent with the 0.003 eV MSE reported in Table 1 for CCSDT as compared to CC4. For the 13 states listed in Table 3, the average correction is -0.011 eV and the average absolute change is 0.022 eV only. More importantly, for the vast majority of cases, the changes are not larger than $\pm 0.03 \mathrm{eV}$, which was the estimated error bar in ref 11.

Nevertheless, a molecule worth discussing is acrolein. For its lowest hallmark $n \rightarrow \pi^{*}$ transition, the CCSD, CC3, and CCSDT VTEs obtained with aug-cc-pVTZ are 3.913, 3.743, and 3.725 eV . The new TBE, 3.72 eV , is consistent with this trend, but is significantly smaller than the original CIPSI-based estimate of 3.78 eV . It is likely that the CIPSI extrapolation error bar was underestimated previously, and we believe that our current TBE is more accurate. Indeed, CC4/6-31+G(d) and CCSDTQ/6-31+G(d) values are very consistent (see Table S7). The second ES, a strongly dipole-allowed ${ }^{1} A^{\prime}(\pi \rightarrow$ $\pi^{*}$ ) transition, was slightly blueshifted in our original work, but the error, -0.02 eV , remains low, a statement also holding for
the higher-lying Rydberg ES of the same symmetry. The original estimate for the second ${ }^{1} A^{\prime \prime}\left(n \rightarrow \pi^{*}\right)$ transition, 6.72 eV , was labeled as unsafe in ref 11 due to its significant double excitation character. Indeed, its percentage of single excitations ( $\% T_{1}$ ) is only $79.4 \%$ according to CC3/aug-cc-pVTZ. Nevertheless, the CC4 correction obtained with aug-ccpVDZ is not very large, and one can likely claim that the new TBE of 6.69 eV stands as the most trustworthy estimate published to date for this particular ES. The higher-lying ${ }^{1} A^{\prime}(\pi$ $\rightarrow \pi^{*}$ ) transition of acrolein has a nature similar to the famous $A_{g}$ state of butadiene with a $\% T_{1}$ of $75 \%$. At the CIPSI/6$31+G(\mathrm{~d})$ level, a value of $8.00 \pm 0.03 \mathrm{eV}$ was obtained previously, ${ }^{29}$ and our CC4 value obtained with the same basis set is consistent with this result: 8.035 eV (see Table S7 in the SI). Based on CC4/aug-cc-pVDZ, i.e., using eq 1 a the new TBE is 7.93 eV , upshifted by 0.06 eV as compared to the original one. Given that for the similar transition in butadiene, the changes between CC4 and CCSDTQ are trifling (Table S7), we consider this new TBE as more trustworthy than the original one, though giving a reliable error bar is not straightforward. Finally, one can also note that, for carbonylfluoride, the current TBE is 0.02 eV smaller than the original CIPSI-based one, for which the estimated extrapolation error bar was 0.02 eV . ${ }^{11}$
4.3. Larger Systems. For the largest systems of Figure 1, CC4 has the advantage to provide a systematic path toward high-accuracy for systems that are beyond reach at the CCSDTQ level. Let us first illustrate this with a set of 30 ESs extracted from systems with five (nonhydrogen) atom (Table 4). To the best of our knowledge, for all five molecules, our results provide the first VTEs including quadruples. As one can see, the differences with the previous TBEs are small and are typically slightly negative, confirming the outcome of the previous section showing that CCSDT/aug-cc-pVTZ yields slightly too large VTEs. The average absolute correction for the data of Table 4 is nonetheless very small, 0.02 eV , which confirms that quality of the original TBEs from the QUEST database.

Let us now take a look at the most problematic cases. Unsurprisingly, the ${ }^{1} A_{1}\left(\pi \rightarrow \pi^{*}\right)$ transition of cyclopentadiene is significantly red-shifted by the CC4 correction $(-0.07 \mathrm{eV})$. This transition was previously classified as unsafe, ${ }^{11}$ due to its $\% T_{1}$ value of $78.9 \%$ and its similar nature to the famous ${ }^{1} A_{g}$ ES of butadiene. For the latter compound, the difference between the CIPSI $/ 6-31+G(\mathrm{~d})$ and CCSDT $/ 6-31+\mathrm{G}(\mathrm{d})$ estimates is $-0.08 \mathrm{eV},{ }^{29}$ which suggests that the CC4 correction of -0.07 eV for the ${ }^{1} A_{1}$ ES of cyclopentadiene is very reasonable. The lowest $B_{2} \mathrm{ES}$ of thiophene is subject to a -0.04 eV shift despite having a very large single excitation character $\left(\% T_{1}=91.5 \%\right)$. All other ESs are even less affected by the CC4 corrections with variations of $\pm 0.03 \mathrm{eV}$ at most.

We now consider three highly symmetric six-membered rings, namely benzene, pyrazine, and tetrazine, for which CC4/ $6-31+G(d)$ calculations are still doable. The results obtained for a large number of ESs are listed in Table 5. For these three compounds, this table is, as far as we are aware of, the first to propose CC-based VTEs including corrections from iterative quadruples.

For benzene, we consider three valence and four Rydberg ESs. These ESs have a very strong single excitation character with $\% T_{1}>92 \%$, except for the lowest ${ }^{1} B_{2 u}$ transition ( $\% T_{1}=$ $86 \%)$. In all cases, one finds that $\mathrm{CC} 4 / 6-31+\mathrm{G}(\mathrm{d})$ provides

Table 4. TBE/aug-cc-pVTZ (in eV) Established for Systems Containing Five Nonhydrogen Atoms ${ }^{a}$

| molecule | transition | TBEs |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | nature | new | prev. | diff. |
| cyclopentadiene | ${ }^{1} B_{2}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 5.54 | $5.56{ }^{\text {b }}$ | -0.02 |
|  | ${ }^{1} A_{2}(\pi \rightarrow 3 s)$ | Ryd | 5.78 | 5.78 | 0.00 |
|  | ${ }^{1} B_{1}(\pi \rightarrow 3 p)$ | Ryd | 6.41 | 6.41 | 0.00 |
|  | ${ }^{1} A_{2}(\pi \rightarrow 3 p)$ | Ryd | 6.45 | 6.46 | -0.01 |
|  | ${ }^{1} B_{2}(\pi \rightarrow 3 p)$ | Ryd | 6.56 | 6.56 | 0.00 |
|  | ${ }^{1} A_{1}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 6.45 | $6.52{ }^{\text {c }}$ | -0.07 |
| difluorodiazirine | ${ }^{1} B_{1}\left(n \rightarrow \pi^{*}\right)$ | Val | 3.73 | 3.74 | -0.01 |
|  | ${ }^{1} A_{2}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 6.99 | 7.00 | -0.01 |
|  | ${ }^{1} B_{2}$ (n.d.) ${ }^{d}$ | Ryd | 8.50 | 8.52 | -0.02 |
| furan | ${ }^{1} A_{2}(\pi \rightarrow 3 s)$ | Ryd | 6.10 | 6.09 | +0.01 |
|  | ${ }^{1} B_{2}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 6.35 | 6.37 | -0.02 |
|  | ${ }^{1} A_{1}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 6.53 | 6.56 | -0.03 |
|  | ${ }^{1} B_{1}(\pi \rightarrow 3 p)$ | Ryd | 6.65 | 6.64 | +0.01 |
|  | ${ }^{1} A_{2}(\pi \rightarrow 3 p)$ | Ryd | 6.82 | 6.81 | +0.01 |
|  | ${ }^{1} B_{2}(\pi \rightarrow 3 p)$ | Ryd | 7.25 | 7.24 | +0.01 |
| pyrrole | ${ }^{1} A_{2}(\pi \rightarrow 3 s)$ | Ryd | 5.23 | 5.24 | -0.01 |
|  | ${ }^{1} B_{1}$ (n.d.) | Ryd | 5.97 | 6.00 | -0.03 |
|  | ${ }^{1} A_{2}(\pi \rightarrow 3 p)$ | Ryd | 6.01 | 6.00 | +0.01 |
|  | ${ }^{1} B_{1}$ (n.d.) ${ }^{e}$ | Ryd | 6.09 |  |  |
|  | ${ }^{1} B_{2}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 6.24 | 6.26 | -0.02 |
|  | ${ }^{1} A_{1}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 6.27 | 6.30 | -0.03 |
|  | ${ }^{1} B_{2}(\pi \rightarrow 3 p)$ | Ryd | 6.82 | 6.83 | -0.01 |
| thiopehene | ${ }^{1} A_{1}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 5.62 | 5.64 | -0.02 |
|  | ${ }^{1} B_{2}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 5.94 | 5.98 | -0.04 |
|  | ${ }^{1} A_{2}(\pi \rightarrow 3 s)$ | Ryd | 6.13 | 6.14 | -0.01 |
|  | ${ }^{1} B_{1}(\pi \rightarrow 3 p)$ | Ryd | 6.12 | 6.14 | -0.02 |
|  | ${ }^{1} A_{2}(\pi \rightarrow 3 p)$ | Ryd | 6.23 | 6.21 | +0.02 |
|  | ${ }^{1} B_{1}(\pi \rightarrow 3 s)$ | Ryd | 6.49 | 6.49 | 0.00 |
|  | ${ }^{1} B_{2}(\pi \rightarrow 3 p)^{f}$ | Ryd | 7.27 | 7.29 | -0.02 |
|  | ${ }^{1} A_{1}\left(\pi \rightarrow \pi^{*}\right)^{f}$ | Val | 7.30 | $7.31{ }^{\text {c }}$ | -0.01 |

a"Val" and "Ryd" stand for valence and Rydberg ESs. TBEs established with eq 1 b , except for difluorodiazirine and furan for which eq 1a is used. Raw data are given in Table S10 in the SI. The previous TBEs taken from refs 11 and 30 are also listed. ${ }^{b}$ The most recent TBE, based on a basis set corrected CIPSI/6-31+G(d) value, is 5.54 eV ; see ref $33 .{ }^{c}$ Considered as unsafe in ref 11 . ${ }^{d}$ Incorrectly labeled as valence in ref $30 .{ }^{e}$ Not considered previously. ${ }^{{ }^{\prime}}$ Nonnegligible valence/Rydberg mixing.
small negative or null corrections, with a maximal amplitude of -0.02 eV , even for the lowest-energy transition.
Eleven ESs of pyrazine are listed in Table 5, including four Rydberg states. The corrections to previous CCSDT-based TBEs are either positive, null, or negative, but most are within the expected error bar of the original estimates. Nevertheless, for the lowest $1^{1} B_{1 g}$ ES, corresponding to a $n \rightarrow \pi^{*}$ excitation, a relatively large correction of -0.04 eV is noticed. This transition has a rather low $\% T_{1}$ value of $84 \%$, which however does not translate into a large CC3/CCSDT difference (ca. 0.01 eV for the three basis sets, see the SI). Likewise the highest-lying $1^{1} B_{1 u}$ ES also undergoes a -0.04 eV correction, but this state was rated unsafe previously, ${ }^{11,33}$ due to an unusually large CC3/CCSDT difference. Four other ESs have their VTEs corrected by $\pm 0.03 \mathrm{eV}$ by CC4, a shift corresponding to the expected error bar of the original TBEs.
In tetrazine, we consider 12 ESs , including two with a pure double-excitation character, for which NEVPT2 was employed

Table 5. TBE/aug-cc-pVTZ (in eV) Established for Benzene, Pyrazine, and Tetrazine ${ }^{a}$

| molecule | transition | TBEs |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | nature | new | prev. | diff. |
| benzene | ${ }^{1} B_{2 u}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 5.05 | 5.06 | -0.01 |
|  | ${ }^{1} B_{1 u}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 6.43 | 6.45 | -0.02 |
|  | ${ }^{1} E_{1 g}(\pi \rightarrow 3 s)$ | Ryd | 6.52 | 6.52 | 0.00 |
|  | ${ }^{1} A_{2 u}(\pi \rightarrow 3 p)$ | Ryd | 7.08 | 7.08 | 0.00 |
|  | ${ }^{1} E_{2 u}(\pi \rightarrow 3 p)$ | Ryd | 7.15 | 7.15 | 0.00 |
|  | ${ }^{1} A_{1 u}(\pi \rightarrow 3 p)^{b}$ | Ryd | 7.23 |  |  |
|  | ${ }^{1} E_{1 u}\left(\pi \rightarrow \pi^{*}\right)^{b}$ | Val | 7.17 |  |  |
| pyrazine | ${ }^{1} B_{3 u}\left(n \rightarrow \pi^{*}\right)$ | Val | 4.14 | 4.15 | -0.01 |
|  | ${ }^{1} A_{u}\left(n \rightarrow \pi^{*}\right)$ | Val | 4.97 | 4.98 | -0.01 |
|  | ${ }^{1} B_{2 u}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 4.99 | 5.02 | -0.03 |
|  | ${ }^{1} B_{2 g}\left(n \rightarrow \pi^{*}\right)$ | Val | 5.68 | 5.71 | -0.03 |
|  | ${ }^{1} A_{g}(n \rightarrow 3 s)$ | Ryd | 6.66 | 6.65 | +0.01 |
|  | ${ }^{1} B_{1 g}\left(n \rightarrow \pi^{*}\right)$ | Val | 6.70 | 6.74 | -0.04 |
|  | ${ }^{1} B_{1 u}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 6.85 | 6.88 | -0.03 |
|  | ${ }^{1} B_{1 g}(\pi \rightarrow 3 s)$ | Ryd | 7.20 | 7.21 | -0.01 |
|  | ${ }^{1} B_{2 u}(n \rightarrow 3 p)$ | Ryd | 7.27 | 7.24 | +0.03 |
|  | ${ }^{1} B_{1 u}(n \rightarrow 3 p)$ | Ryd | 7.45 | 7.44 | +0.01 |
|  | ${ }^{1} B_{1 u}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 7.94 | $7.98{ }^{\text {c }}$ | -0.04 |
| tetrazine | ${ }^{1} B_{3 u}\left(n \rightarrow \pi^{*}\right)$ | Val | 2.46 | 2.47 | -0.01 |
|  | ${ }^{1} A_{u}\left(n \rightarrow \pi^{*}\right)$ | Val | 3.68 | 3.69 | -0.01 |
|  | ${ }^{1} A_{g}\left(n, n \rightarrow \pi^{*}, \pi^{*}\right)$ | Val |  | $4.61{ }^{\text {d }}$ |  |
|  | ${ }^{1} B_{1 g}\left(n \rightarrow \pi^{*}\right)$ | Val | 4.87 | 4.93 | -0.06 |
|  | ${ }^{1} B_{2 u}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 5.17 | 5.21 | -0.04 |
|  | ${ }^{1} B_{2 g}\left(n \rightarrow \pi^{*}\right)$ | Val | 5.50 | 5.45 | -0.05 |
|  | ${ }^{1} A_{u}\left(n \rightarrow \pi^{*}\right)$ | Val | 5.51 | 5.53 | -0.02 |
|  | ${ }^{1} B_{3 g}\left(n, n \rightarrow \pi^{*}, \pi^{*}\right)$ | Val |  | $6.15{ }^{\text {d }}$ |  |
|  | ${ }^{1} B_{2 g}\left(n \rightarrow \pi^{*}\right)$ | Val | 6.05 | 6.12 | -0.07 |
|  | ${ }^{1} B_{3 g}(n \rightarrow 3 s)^{b}$ | Ryd | 6.47 |  |  |
|  | ${ }^{1} B_{3 u}\left(\pi \rightarrow \pi^{*}\right)^{b}$ | Val | 6.67 |  |  |
|  | ${ }^{1} B_{1 g}\left(n \rightarrow \pi^{*}\right)$ | Val | 6.89 | 6.91 | -0.02 |

a"Val" and "Ryd" stand for valence and Rydberg ESs. TBEs established using eq 1 b. Raw data are given in Table S11 in the SI. The previous TBEs taken from ref 11 are also listed. ${ }^{b}$ Not considered previously. ${ }^{c}$ Considered as unsafe in ref $11 .{ }^{d}$ Genuine doubly excited states with TBEs obtained at the NEVPT2/aug-cc-pVTZ level and considered as unsafe in ref 11 .
in the QUEST database to obtain TBEs, whereas, for the other transitions, CCSDT/aug-cc-pVTZ or basis set corrected CCSDT/aug-cc-pVDZ values were originally selected as TBEs. ${ }^{11}$ All the latter were rated as safe as the energy differences between CCSDT and CC3 were smaller than 0.03 eV despite $\% T_{1}$ values often smaller than $90 \%{ }^{11,33}$ For several ESs, small negative CC4 corrections, -0.01 or -0.02 eV , are obtained. For the lowest $\pi \rightarrow \pi^{*}$ transition, the correction is only slightly larger, i.e., -0.04 eV . However, for three $n \rightarrow \pi^{*}$ transitions $\left(1^{1} B_{1 g}, 1^{1} B_{2 g}\right.$, and $\left.2^{1} B_{2 g}\right)$, much larger changes ( $-0.06,-0.05$, and -0.07 eV ) are induced by the inclusion of quadruples, indicating that our original assessment of the TBE quality was optimistic. Interestingly, for these three transitions, one has $\% T_{1}<85 \%$, in contrast with the other singly excited ESs for which $\% T_{1}>85 \%$. This observation suggests that the $85 \%$ barrier might be the limit for which CCSDT can be considered trustworthy, irrespective of the difference between CC3 and CCSDT. For the two doubly excited states of tetrazine listed in Tables 5 and S11 that are both characterized

Table 6. TBE/aug-cc-pVTZ (in eV) Established for Hexatriene ${ }^{a}$

| molecule | transition | TBEs |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | nature | new | prev. | diff. |
| hexatriene | ${ }^{1} B_{u}\left(\pi \rightarrow \pi^{*}\right)$ | Val | 5.34 | 5.37 | -0.03 |
|  | ${ }^{1} A_{g}\left(\pi \rightarrow \pi^{*}\right)$ | Val | $5.46 / 5.43{ }^{\text {c }}$ | $5.62{ }^{\text {b }}$ | -0.16/-0.19 |
|  | ${ }^{1} A_{u}(\pi \rightarrow 3 s)$ | Ryd | 5.79 | 5.79 | 0.00 |
|  | ${ }^{1} B_{g}(\pi \rightarrow 3 p)$ | Ryd | 5.93 | 5.94 | -0.01 |

a"Val" and "Ryd" stand for valence and Rydberg ESs. TBEs established using eq 1d. Raw data are given in Table S11 in the SI. The previous TBEs taken from ref 33 are also listed. ${ }^{b}$ Considered as unsafe in ref 33 . ${ }^{c}$ See text.
by $\% T_{1}<1 \%,{ }^{11}$ the differences between CC4/6-31+G(d) and CCSDT $/ 6-31+\mathrm{G}(\mathrm{d})$ are very large, that is, $-0.79 \mathrm{eV}\left(A_{g}\right)$ and $-0.99 \mathrm{eV}\left(B_{3 g}\right)$. These values are nonetheless typical for these challenging ESs with a dominant contribution from the doubly excited determinants. Applying eq 1 b delivers VTEs of 5.17 and 6.44 eV . These two results logically remain higher in energy than the corresponding NEVPT2/aug-cc-pVTZ values of 4.61 and 6.15 eV , which we consider as the most accurate TBEs available to date, though with an error bar of approximately $\pm 0.1 \mathrm{eV}$.
Finally, let us briefly discuss the challenging case of hexatriene (Table 6). The original TBEs for this small polyene have been obtained via eq 2a: 5.37, 5.62, 5.79, and 5.94 eV for the lowest ${ }^{1} B_{u},{ }^{1} A_{g},{ }^{1} A_{u}$, and ${ }^{1} B_{g}$ ESs, respectively. ${ }^{33}$ All were considered safe, except the second one that has a very significant contribution from the double excitations with $\% T_{1}=$ $65 \%$ (i.e., $10 \%$ less than in butadiene). Using eq 1 d to include CC4 corrections leads to improved TBEs of 5.34, 5.46, 5.79, and 5.93 eV . Only the ${ }^{1} A_{g}$ VTE undergoes a significant downshift $(-0.16 \mathrm{eV})$ due to the quadruples. Logically, this 5.46 eV value can be considered as an upper bound, and we roughly approximate the exact value to be of the order of 5.43 eV . To support this crude estimate, we looked at the equivalent transition in the smaller polyene, butadiene. Indeed, in butadiene, the difference between the $\mathrm{CC} 4 / 6-31+\mathrm{G}(\mathrm{d})$ and FCI/ $6-31+\mathrm{G}(\mathrm{d})$ values is -0.01 eV ; the correction should be larger for hexatriene given its smaller $\% T_{1}$ value. In addition, as can be seen in ref 29 , the NEVPT2/6-31+G(d) value is 0.20 eV larger than the $\mathrm{FCI} / 6-31+\mathrm{G}(\mathrm{d})$ result for butadiene, and applying such rigid shift to NEVPT2/aug-cc-pVTZ VTE of hexatriene ${ }^{29}$ would yield an estimate of 5.44 eV . Although it would not be suited to rate this new TBE/aug-cc-pVTZ of 5.43 eV as safe, it is likely one of the most accurate estimate published to date for this dark transition.

## 5. CONCLUSIONS AND OUTLOOK

Three main problems were tackled in the present study. Our first aim was to confirm that CC4 provides very accurate VTEs. To this end, we considered 43 molecules containing between two and four (nonhydrogen) atoms and computed their ESs with three Gaussian basis sets containing diffuse functions, i.e., $6-31+G(d)$, aug-cc-pVDZ, and aug-cc-pVTZ. We started by defining more than 200 CCSDTQ reference values. Excluding the few pathological pure doubly excited states of $(n, n) \rightarrow$ $\left(\pi^{*}, \pi^{*}\right)$ nature but conserving all other transitions that have a nonnegligible double excitation character ( $2^{1} A_{g}$ in butadiene, $2^{1} A^{\prime \prime}$ in acrolein, etc), we showed that CC4 is an excellent approximation of CCSDTQ with a mean absolute error of 0.003 eV . This is a totally negligible deviation for the vast majority of chemical studies ( $<0.1 \mathrm{kcal} \cdot \mathrm{mol}^{-1}$ ). We also
showed that neither CC3 nor CCSDT could deliver the same level of accuracy for the very same set of ESs.

Second, we investigated the performance of additive basis set correction schemes in the EOM-CC context using the same set of ESs as in the first part. It appeared that these popular correction strategies indeed statistically improve the quality of the VTEs. More interestingly, such basis set corrections were found particularly powerful when starting from CC3/aug-ccpVTZ or CCSDT/aug-cc-pVTZ, as they allowed estimating the true CC4/aug-cc-pVTZ or CCSDTQ/aug-cc-pVTZ VTEs with average errors of approximately 0.005 eV . Of course, it is essential to confirm these trends for larger compounds, but the cost of CC4/aug-cc-pVTZ and/or CCSDTQ/aug-cc-pVTZ unfortunately prevents us from doing it as of today.

Finally, CC4 was employed to improve previous CCSDTbased TBEs of the QUEST database. ${ }^{33}$ For benzene, cyclopentadiene, difluorodiazirine, furan, hexatriene, pyrazine, pyrrole, tetrazine, and thiophene, it was possible to perform CC4/6-31+G(d) and/or CC4/aug-cc-pVDZ calculations, whereas CCSDTQ remains beyond reach. The corrections obtained were, in most cases, smaller than the expected average error of the original TBEs (ca. $\pm 0.03 \mathrm{eV}$ ). Nevertheless, most corrections were null or negative, hinting at a slight overestimation trend in our original TBEs. In addition, some of the estimates originally viewed as unsafe can be now considered as trustworthy $\left(2^{1} A^{\prime \prime}\right.$ in acrolein, $2^{1} A_{1}$ in cyclopentadiene, $3^{1} A_{1}$ in thiophene, etc.), whereas a few optimistic assessments have been revised, resulting in more accurate TBEs (in particular for the valence $B_{1 g}$ and $B_{2 g}$ ESs of tetrazine).

We are currently pursuing our efforts to improve the quality, size, and diversity of the QUEST database, so as to provide the most trustworthy reference values possible to the community. We plan to publish, within a year or two, an expanded and improved version of the QUEST database.

## - ASSOCIATED CONTENT

## (s) Supporting Information

The Supporting Information is available free of charge at https://pubs.acs.org/doi/10.1021/acs.jctc.2c00416.

Raw VTEs and geometries (PDF)

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## Notes

The authors declare no competing financial interest.

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## - REFERENCES

(1) Roos, B. O.; Andersson, K.; Fulscher, M. P.; Malmqvist, P.-A.; Serrano-Andrés, L. Multiconfigurational Perturbation Theory: Applications in Electronic Spectroscopy. In Advances in Chemical Physics: New Methods in Computational Quantum Mechanics; Prigogine, I., Rice, S. A., Eds.; Wiley: New York, 1996; Vol. 93, pp 219-331.
(2) Piecuch, P.; Kowalski, K.; Pimienta, I. S. O.; Mcguire, M. J. Recent Advances in Electronic Structure Theory: Method of Moments of Coupled-Cluster Equations and Renormalized Coupled-Cluster Approaches. Int. Rev. Phys. Chem. 2002, 21, 527655.
(3) Dreuw, A.; Head-Gordon, M. Single-Reference ab initio Methods for the Calculation of Excited States of Large Molecules. Chem. Rev. 2005, 105, 4009-4037.
(4) Krylov, A. I. Spin-Flip Equation-of-Motion Coupled-Cluster Electronic Structure Method for a Description of Excited States, Bond Breaking, Diradicals, and Triradicals. Acc. Chem. Res. 2006, 39, 83-91.
(5) Sneskov, K.; Christiansen, O. Excited State Coupled Cluster Methods. WIREs Comput. Mol. Sci. 2012, 2, 566-584.
(6) González, L.; Escudero, D.; Serrano-Andrès, L. Progress and Challenges in the Calculation of Electronic Excited States. ChemPhysChem 2012, 13, 28-51.
(7) Laurent, A. D.; Jacquemin, D. TD-DFT Benchmarks: A Review. Int. J. Quantum Chem. 2013, 113, 2019-2039.
(8) Adamo, C.; Jacquemin, D. The calculations of Excited-State Properties with Time-Dependent Density Functional Theory. Chem. Soc. Rev. 2013, 42, 845-856.
(9) Ghosh, S.; Verma, P.; Cramer, C. J.; Gagliardi, L.; Truhlar, D. G. Combining Wave Function Methods with Density Functional Theory for Excited States. Chem. Rev. 2018, 118, 7249-7292.
(10) Blase, X.; Duchemin, I.; Jacquemin, D.; Loos, P. F. The BetheSalpeter Formalism: From Physics to Chemistry. J. Phys. Chem. Lett. 2020, 11, 7371-7382.
(11) Loos, P.-F.; Lipparini, F.; Boggio-Pasqua, M.; Scemama, A.; Jacquemin, D. A Mountaineering Strategy to Excited States: HighlyAccurate Energies and Benchmarks for Medium Size Molecules. J. Chem. Theory Comput. 2020, 16, 1711-1741.
(12) Pople, J. A.; Head-Gordon, M.; Fox, D. J.; Raghavachari, K.; Curtiss, L. A. Gaussian-1 Theory: A General Procedure for Prediction of Molecular Energies. J. Chem. Phys. 1989, 90, 5622-5629.
(13) Curtiss, L. A.; Raghavachari, K.; Trucks, G. W.; Pople, J. A. Gaussian-2 Theory for Molecular Energies of First- and Second-row Compounds. J. Chem. Phys. 1991, 94, 7221-7230.
(14) Curtiss, L. A.; Raghavachari, K.; Redfern, P. C.; Pople, J. A. Investigation of the Use of B3LYP Zero-Point Energies and Geometries in the Calculation of Enthalpies of Formation. Chem. Phys. Lett. 1997, 270, 419-426.
(15) Curtiss, L. A.; Raghavachari, K.; Redfern, P. C.; Rassolov, V.; Pople, J. A. Gaussian-3 (G3) Theory for Molecules Containing First and Second-Row Atoms. J. Chem. Phys. 1998, 109, 7764-7776.
(16) Curutchet, C.; Scholes, G. D.; Mennucci, B.; Cammi, R. How Solvent Controls Electronic Energy Transfer and Light Harvesting: Toward a Quantum-Mechanical Description of Reaction Field and Screening Effects. J. Phys. Chem. B 2007, 111, 13253-13265.
(17) Tajti, A.; Szalay, P. G.; Császár, A. G.; Kállay, M.; Gauss, J.; Valeev, E. F.; Flowers, B. A.; Vázquez, J.; Stanton, J. F. HEAT: High Accuracy Extrapolated Ab Initio Thermochemistry. J. Chem. Phys. 2004, 121, 11599-11613.
(18) Bomble, Y. J.; Vázquez, J.; Kállay, M.; Michauk, C.; Szalay, P. G.; Császár, A. G.; Gauss, J.; Stanton, J. F. High-Accuracy Extrapolated ab initio Thermochemistry. II. Minor Improvements to the Protocol and a Vital Simplification. J. Chem. Phys. 2006, 125, 064108.
(19) Harding, M. E.; Vazquez, J.; Ruscic, B.; Wilson, A. K.; Gauss, J.; Stanton, J. F. High-Accuracy Extrapolated ab Initio Thermochemistry. III. Additional Improvements and Overview. J. Chem. Phys. 2008, 128, 114111.
(20) van Setten, M. J.; Weigend, F.; Evers, F. The GW-Method for Quantum Chemistry Applications: Theory and Implementation. J. Chem. Theory Comput. 2013, 9, 232-246.
(21) Loos, P. F.; Scemama, A.; Jacquemin, D. The Quest for HighlyAccurate Excitation Energies: A Computational Perspective. J. Phys. Chem. Lett. 2020, 11, 2374-2383.
(22) Serrano-Andrés, L.; Mechán, M.; Nebot-Gil, I.; Lindh, R.; Roos, B. O. Towards an Accurate Molecular Orbital Theory for Excited States: Ethene, Butadiene, and Hexatriene. J. Chem. Phys. 1993, 98, 3151-3162.
(23) Serrano-Andrés, L.; Merchán, M.; Nebot-Gil, I.; Roos, B. O.; Fulscher, M. Theoretical Study of the Electronic Spectra of Cyclopentadiene, Pyrrole, and Furan. J. Am. Chem. Soc. 1993, 115, 6184-6197.
(24) Serrano-Andres, L.; Merchan, M.; Roos, B. O.; Lindh, R. Theoretical Study of the Internal Charge Transfer in Aminobenzonitriles. J. Am. Chem. Soc. 1995, 117, 3189-3204.
(25) Schreiber, M.; Silva-Junior, M. R.; Sauer, S. P. A.; Thiel, W. Benchmarks for Electronically Excited States: CASPT2, CC2, CCSD and CC3. J. Chem. Phys. 2008, 128, 134110.
(26) Silva-Junior, M. R.; Sauer, S. P. A.; Schreiber, M.; Thiel, W. Basis Set Effects on Coupled Cluster Benchmarks of Electronically Excited States: CC3, CCSDR(3) and CC2. Mol. Phys. 2010, 108, 453-465.
(27) Silva-Junior, M. R.; Schreiber, M.; Sauer, S. P. A.; Thiel, W. Benchmarks of Electronically Excited States: Basis Set Effects on CASPT2 Results. J. Chem. Phys. 2010, 133, 174318.
(28) Loos, P.-F.; Scemama, A.; Blondel, A.; Garniron, Y.; Caffarel, M.; Jacquemin, D. A Mountaineering Strategy to Excited States: Highly-Accurate Reference Energies and Benchmarks. J. Chem. Theory Comput. 2018, 14, 4360-4379.
(29) Loos, P.-F.; Boggio-Pasqua, M.; Scemama, A.; Caffarel, M.; Jacquemin, D. Reference Energies for Double Excitations. J. Chem. Theory Comput. 2019, 15, 1939-1956.
(30) Loos, P.-F.; Scemama, A.; Boggio-Pasqua, M.; Jacquemin, D. A Mountaineering Strategy to Excited States: Highly-Accurate Energies and Benchmarks for Exotic Molecules and Radicals. J. Chem. Theory Comput. 2020, 16, 3720-3736.
(31) Chrayteh, A.; Blondel, A.; Loos, P.-F.; Jacquemin, D. A Mountaineering Strategy to Excited States: Highly-Accurate Oscillator Strengths and Dipole Moments of Small Molecules. J. Chem. Theory Comput. 2021, 17, 416-438.
(32) Loos, P.-F.; Comin, M.; Blase, X.; Jacquemin, D. Reference Energies for Intramolecular Charge-Transfer Excitations. J. Chem. Theory Comput. 2021, 17, 3666-3686.
(33) Véril, M.; Scemama, A.; Caffarel, M.; Lipparini, F.; BoggioPasqua, M.; Jacquemin, D.; Loos, P.-F. QUESTDB: a Database of Highly-Accurate Excitation Energies for the Electronic Structure Community. WIREs Comput. Mol. Sci. 2021, 11, No. el517.
(34) Huron, B.; Malrieu, J. P.; Rancurel, P. Iterative Perturbation Calculations of Ground and Excited State Energies from Multiconfigurational Zeroth-Order Wavefunctions. J. Chem. Phys. 1973, 58, 5745-5759.
(35) Giner, E.; Scemama, A.; Caffarel, M. Using Perturbatively Selected Configuration Interaction in Quantum Monte Carlo Calculations. Can. J. Chem. 2013, 91, 879-885.
(36) Giner, E.; Scemama, A.; Caffarel, M. Fixed-Node Diffusion Monte Carlo Potential Energy Curve of the Fluorine Molecule $\mathrm{F}_{2}$ Using Selected Configuration Interaction Trial Wavefunctions. J. Chem. Phys. 2015, 142, 044115.
(37) Garniron, Y.; Scemama, A.; Loos, P.-F.; Caffarel, M. Hybrid Stochastic-Deterministic Calculation of the Second-Order Perturbative Contribution of Multireference Perturbation Theory. J. Chem. Phys. 2017, 147, 034101.
(38) Garniron, Y.; Scemama, A.; Giner, E.; Caffarel, M.; Loos, P.-F. Selected Configuration Interaction Dressed by Perturbation. J. Chem. Phys. 2018, 149, 064103.
(39) Garniron, Y.; Applencourt, T.; Gasperich, K.; Benali, A.; Ferté, A.; Paquier, J.; Pradines, B.; Assaraf, R.; Reinhardt, P.; Toulouse, J.; Barbaresco, P.; Renon, N.; David, G.; Malrieu, J.-P.; Véril, M.; Caffarel, M.; Loos, P.-F.; Giner, E.; Scemama, A. Quantum Package 2.0: An Open-Source Determinant-Driven Suite of Programs. J. Chem. Theory Comput. 2019, 15, 3591-3609.
(40) Kucharski, S. A.; Bartlett, R. J. Recursive Intermediate Factorization and Complete Computational Linearization of the Coupled-Cluster Single, Double, Triple, and Quadruple Excitation Equations. Theor. Chim. Acta 1991, 80, 387-405.
(41) Noga, J.; Bartlett, R. J. The Full CCSDT Model for Molecular Electronic Structure. J. Chem. Phys. 1987, 86, 7041-7050.
(42) Scuseria, G. E.; Schaefer, H. F. A New Implementation of the Full CCSDT Model for Molecular Electronic Structure. Chem. Phys. Lett. 1988, 152, 382-386.
(43) Kucharski, S. A.; Włoch, M.; Musiał, M.; Bartlett, R. J. CoupledCluster Theory for Excited Electronic States: The Full Equation-OfMotion Coupled-Cluster Single, Double, and Triple Excitation Method. J. Chem. Phys. 2001, 115, 8263-8266.
(44) Kowalski, K.; Piecuch, P. The Active-Space Equation-of-Motion Coupled-Cluster Methods for Excited Electronic States: Full EOMCCSDt. J. Chem. Phys. 2001, 115, 643-651.
(45) Kowalski, K.; Piecuch, P. Excited-State Potential Energy Curves of $\mathrm{CH}^{+}$: a Comparison of the EOMCCSDt And Full EOMCCSDT Results. Chem. Phys. Lett. 2001, 347, 237-246.
(46) Loos, P.-F.; Jacquemin, D. Is $\operatorname{ADC}(3)$ as Accurate as CC3 for Valence and Rydberg Transition Energies? J. Phys. Chem. Lett. 2020, 11, 974-980.
(47) Sarkar, R.; Loos, P.-F.; Boggio-Pasqua, M.; Jacquemin, D. Assessing the Performances of CASPT2 and NEVPT2 for Vertical Excitation Energies. J. Chem. Theory Comput. 2022, 18, 2418-2436.
(48) Giner, E.; Scemama, A.; Toulouse, J.; Loos, P. F. Chemically Accurate Excitation Energies With Small Basis Sets. J. Chem. Phys. 2019, 151, 144118.
(49) Otis, L.; Craig, I.; Neuscamman, E. A hybrid approach to excited-state-specific variational Monte Carlo and doubly excited states. J. Chem. Phys. 2020, 153, 234105.
(50) Loos, P.-F.; Blase, X. Dynamical correction to the BetheSalpeter equation beyond the plasmon-pole approximation. J. Chem. Phys. 2020, 153, 114120.
(51) Kossoski, F.; Marie, A.; Scemama, A.; Caffarel, M.; Loos, P.-F. Excited States from State-Specific Orbital-Optimized Pair Coupled Cluster. J. Chem. Theory Comput. 2021, 17, 4756.
(52) Gould, T.; Kronik, L.; Pittalis, S. Double excitations in molecules from ensemble density functionals: Theory and approximations. Phys. Rev. A 2021, 104, 022803.
(53) Gould, T.; Hashimi, Z.; Kronik, L.; Dale, S. G. Single Excitation Energies Obtained from the Ensemble "HOMO-LUMO Gap": Exact Results and Approximations. J. Phys. Chem. Lett. 2022, 13, 24522458.
(54) Grotjahn, R.; Kaupp, M. Assessment of Hybrid Functionals for Singlet and Triplet Eccitations: Why do Some Local Hybrid Functionals Perform so well for Triplet Excitation Energies? J. Chem. Phys. 2021, 155, 124108.
(55) Liang, J.; Feng, X.; Hait, D.; Head-Gordon, M. Revisiting the performance of time-dependent density functional theory for electronic excitations: Assessment of 43 popular and recently developed functionals from rungs one to four. J. Chem. Theory. Comput. 2022, 18, 3460.
(56) Casanova-Páez, M.; Dardis, M. B.; Goerigk, L. $\omega$ B2PLYP and $\omega$ B2GPPLYP: The First Two Double-Hybrid Density Functionals with Long-Range Correction Optimized for Excitation Energies. J. Chem. Theory Comput. 2019, 15, 4735-4744.
(57) Mester, D.; Kállay, M. A Simple Range-Separated DoubleHybrid Density Functional Theory for Excited States. J. Chem. Theory Comput. 2021, 17, 927-942.
(58) Casanova-Páez, M.; Goerigk, L. Time-Dependent Long-RangeCorrected Double-Hybrid Density Functionals with Spin-Component and Spin-Opposite Scaling: A Comprehensive Analysis of SingletSinglet and Singlet-Triplet Excitation Energies. J. Chem. Theory. Comput. 2021, 17, 5165-5186.
(59) Mester, D.; Kállay, M. Spin-Scaled Range-Separated DoubleHybrid Density Functional Theory for Excited States. J. Chem. Theory Comput. 2021, 17, 4211-4224.
(60) Hait, D.; Head-Gordon, M. Excited State Orbital Optimization via Minimizing the Square of the Gradient: General Approach and Application to Singly and Doubly Excited States via Density Functional Theory. J. Chem. Theory Comput. 2020, 16, 1699-1710.
(61) Hait, D.; Head-Gordon, M. Orbital Optimized Density Functional Theory for Electronic Excited States. J. Phys. Chem. Lett. 2021, 12, 4517-4529.
(62) Kállay, M.; Gauss, J. Approximate Treatment of Higher Excitations in Coupled-Cluster Theory. J. Chem. Phys. 2005, 123, 214105.
(63) Loos, P.-F.; Matthews, D. A.; Lipparini, F.; Jacquemin, D. How Accurate are EOM-CC4 Vertical Excitation Energies? J. Chem. Phys. 2021, 154, 221103.
(64) Stanton, J. F.; Gauss, J.; Cheng, L.; Harding, M. E.; Matthews, D. A.; Szalay, P. G.; Auer, A. A.; Bartlett, R. J.; Benedikt, U.; Berger, C.; Bernholdt, D. E.; Bomble, Y. J.; Christiansen, O.; Engel, F.; Faber, R.; Heckert, M.; Heun, O.; Hilgenberg, M.; Huber, C.; Jagau, T.-C.; Jonsson, D.; Jusélius, J.; Kirsch, T.; Klein, K.; Lauderdale, W. J.; Lipparini, F.; Metzroth, T.; Mück, L. A.; O’Neill, D. P.; Price, D. R.; Prochnow, E.; Puzzarini, C.; Ruud, K.; Schiffmann, F.; Schwalbach, W.; Simmons, C.; Stopkowicz, S.; Tajti, A.; Vázquez, J.; Wang, F.; Watts, J. D. Cfour (v 2.1): Coupled-Cluster Techniques for Computational Chemistry, a Quantum-Chemical Program Package; http://www. cfour.de (accessed 2022-05-05).
(65) Matthews, D. A.; Cheng, L.; Harding, M. E.; Lipparini, F.; Stopkowicz, S.; Jagau, T.-C.; Szalay, P. G.; Gauss, J.; Stanton, J. F. Coupled-Cluster Techniques for Computational Chemistry: The CFOUR Program Package. J. Chem. Phys. 2020, 152, 214108.
(66) Matthews, D. A.; Stanton, J. F. Non-Orthogonal SpinAdaptation of Coupled Cluster Methods: A new Implementation of Methods Including Quadruple Excitations. J. Chem. Phys. 2015, 142, 064108.
(67) Matthews, D. A.; Stanton, J. F. In Developments in Physical \& Theoretical Chemistry: Mathematical Physics in Theoretical Chemistry; Blinder, S., House, J., Eds.; Elsevier: Amsterdam, 2019; Chapter 10, pp 327-375.
(68) Christiansen, O.; Koch, H.; Jørgensen, P. The Second-Order Approximate Coupled Cluster Singles and Doubles Model CC2. Chem. Phys. Lett. 1995, 243, 409-418.
(69) Hättig, C.; Weigend, F. CC2 Excitation Energy Calculations on Large Molecules Using the Resolution of the Identity Approximation. J. Chem. Phys. 2000, 113, 5154-5161.
(70) Purvis, G. P., III; Bartlett, R. J. A Full Coupled-Cluster Singles and Doubles Model: The Inclusion of Disconnected Triples. J. Chem. Phys. 1982, 76, 1910-1918.
(71) Scuseria, G. E.; Scheiner, A. C.; Lee, T. J.; Rice, J. E.; Schaefer, H. F. The Closed-Shell Coupled Cluster Single and Double Excitation (CCSD) Model for the Description of Electron Correlation. A Comparison with Configuration Interaction (CISD) Results. J. Chem. Phys. 1987, 86, 2881-2890.
(72) Koch, H.; Jensen, H. J. A.; Jorgensen, P.; Helgaker, T. Excitation Energies from the Coupled Cluster Singles and Doubles Linear Response Function (CCSDLR). Applications to $\mathrm{Be}, \mathrm{CH}^{+}, \mathrm{CO}$, and $\mathrm{H}_{2} \mathrm{O}$. J. Chem. Phys. 1990, 93, 3345-3350.
(73) Stanton, J. F.; Bartlett, R. J. The Equation of Motion CoupledCluster Method - A Systematic Biorthogonal Approach to Molecular Excitation Energies, Transition-Probabilities, and Excited-State Properties. J. Chem. Phys. 1993, 98, 7029-7039.
(74) Stanton, J. F. Many-Body Methods for Excited State Potential Energy Surfaces. I: General Theory of Energy Gradients for the Equation-of-Motion Coupled-Cluster Method. J. Chem. Phys. 1993, 99, 8840-8847.
(75) Christiansen, O.; Koch, H.; Jørgensen, P. Response Functions in the CC3 Iterative Triple Excitation Model. J. Chem. Phys. 1995, 103, 7429-7441.
(76) Koch, H.; Christiansen, O.; Jorgensen, P.; Olsen, J. Excitation Energies of $\mathrm{BH}, \mathrm{CH}_{2}$ and Ne in Full Configuration Interaction and the Hierarchy CCS, CC2, CCSD and CC3 of Coupled Cluster Models. Chem. Phys. Lett. 1995, 244, 75-82.
(77) Koch, H.; Christiansen, O.; Jorgensen, P.; Sanchez de Merás, A. M.; Helgaker, T. The CC3Model: An Iterative Coupled Cluster Approach Including Connected Triples. J. Chem. Phys. 1997, 106, 1808-1818.
(78) Kállay, M.; Gauss, J. Calculation of Excited-State Properties Using General Coupled-Cluster and Configuration-Interaction Models. J. Chem. Phys. 2004, 121, 9257-9269.
(79) Kállay, M.; Rolik, Z.; Csontos, J.; Nagy, P.; Samu, G.; Mester, D.; Csóka, J.; Szabó, B.; Ladjánszki, I.; Szegedy, L.; Ladóczki, B.; Petrov, K.; Farkas, M.; Mezei, P. D.; Hégely, B. MRCC, Quantum Chemical Program, 2017; http://www.mrcc.hu (accessed 2022-0505).
(80) Kallay, M.; Nagy, P. E.; Mester, D.; Rolik, Z.; Samu, J.; Ans Csontos, G.; Csoka, J.; Szabo, P. B.; Gyevi-Nagu, L.; Hegely, B.; Ladjanski, I.; Szegedy, L.; Ladoczki, B.; Petrov, K.; Farkas, M.; Mezei, P. D.; Ganyecz, A. The MRCC Program System: Accurate Quantum Chemistry from Water to Proteins. J. Chem. Phys. 2020, 152, 074107.
(81) Kánnár, D.; Szalay, P. G. Benchmarking Coupled Cluster Methods on Valence Singlet Excited States. J. Chem. Theory Comput. 2014, 10, 3757-3765.
(82) Tajti, A.; Szalay, P. G. Accuracy of Spin-Component-Scaled CC2 Excitation Energies and Potential Energy Surfaces. J. Chem. Theory Comput. 2019, 15, 5523-5531.
(83) Balabanov, N. B.; Peterson, K. A. Basis set Limit Electronic Excitation Energies, Ionization Potentials, and Electron Affinities for the 3d Transition Metal Atoms: Coupled Cluster and Multireference Methods. J. Chem. Phys. 2006, 125, 074110.
(84) Kamiya, M.; Hirata, S. Higher-Order Equation-of-Motion Coupled-Cluster Methods for Ionization Processes. J. Chem. Phys. 2006, 125, 074111.
(85) Peach, M. J. G.; Tozer, D. J. Overcoming Low Orbital Overlap and Triplet Instability Problems in TDDFT. J. Phys. Chem. A 2012, 116, 9783-9789.
(86) Watson, M. A.; Chan, G. K.-L. Excited States of Butadiene to Chemical Accuracy: Reconciling Theory and Experiment. J. Chem. Theory Comput. 2012, 8, 4013-4018.
(87) Feller, D.; Peterson, K. A.; Davidson, E. R. A Systematic Approach to Vertically Excited States of Ethylene Using Configuration Interaction and Coupled Cluster Techniques. J. Chem. Phys. 2014, 141, 104302.
(88) Gui, X.; Holzer, C.; Klopper, W. Accuracy Assessment of GW Starting Points for Calculating Molecular Excitation Energies Using the Bethe-Salpeter Formalism. J. Chem. Theory Comput. 2018, 14, 2127-2136.
(89) Franke, P. R.; Moore, K. B.; Schaefer, H. F.; Douberly, G. E. tert-Butyl Peroxy Radical: Ground and First Excited State Energetics and Fundamental Frequencies. Phys. Chem. Chem. Phys. 2019, 21, 9747-9758.


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