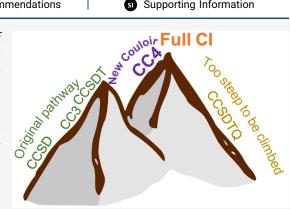
A Mountaineering Strategy to Excited States: Revising Reference Values with EOM-CC4

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Cite This: J. Chem. Theory Comput. 2022, 18, 4418–4427 ACCESS I II Metrics & More III Article Recommendations Supporting Information ABSTRACT: In the framework of the computational determination of highly accurate vertical excitation energies in small organic compounds, we explore the possibilities offered by the equation-of-motion formalism relying on the approximate fourth-order coupled-cluster (CC) method,

relying on the approximate fourth-order coupled-cluster (CC) method, CC4. We demonstrate, using an extended set of more than 200 reference values based on CC including up to quadruples excitations (CCSDTQ), that CC4 is an excellent approximation to CCSDTQ for excited states with a dominant contribution from single excitations with an average deviation as small as 0.003 eV. We next assess the accuracy of several additive basis set correction schemes, in which vertical excitation energies obtained with a compact basis set and a high-order CC method are corrected with lowerorder CC calculations performed in a larger basis set. Such strategies are found to be overall very beneficial, though their accuracy depends



significantly on the actual scheme. Finally, CC4 is employed to improve several theoretical best estimates of the QUEST database for molecules containing between four and six (nonhydrogen) atoms, for which previous estimates were computed at the CCSDT level.

1. INTRODUCTION

Defining sets of high-quality reference values that can be employed to reliably assess the pros and cons of lower-cost theoretical methods is a very popular and useful research line in quantum chemistry.¹⁻¹¹ Although experimental values may constitute natural references for some properties (e.g., thermodynamical and kinetic data), $^{12-16}$ it is often welcome to rely on state-of-the-art electronic structure methods to produce reference values for other properties. Of course, this latter approach is intrinsically limited by the computational cost of these high-accuracy models. However, it has the undeniable advantage to allow well-grounded comparisons within a unique, well-defined set of parameters.¹⁷⁻²⁰ Indeed, one can perform comparisons with exactly the same geometries, basis set, solvent model, etc. Purely theoretical reference values are especially useful for electronic excited states (ESs), as the most straightforwardly accessible theoretical values, namely, vertical transition energies (VTEs), are not directly measurable experimentally.²¹ This explains why the determination of accurate VTEs has been an active and productive avenue of research during the past three decades, with, in particular, valuable works from $Roos'^{22-24}$ and Thiel's²⁵⁻²⁷ groups.

Since 2018, our groups have made several contributions in this field,^{11,28-32} and it eventually led to the creation of the QUEST database (see https://lcpq.github.io/QUESTDB_website) that contains a large panel of reference VTEs for molecules containing from 1 to 10 nonhydrogen atoms.³³ At

the present stage, the QUEST database includes more than 500 theoretical best estimates (TBEs) for diverse ESs (singlet, doublet, and triplet; valence and Rydberg; charge transfer, singly- and doubly excited states) that have been established with the aug-cc-pVTZ basis set. Typically, the TBEs contained in the QUEST database are produced using VTEs computed with the selected configuration interaction algorithm named "Configuration Interaction using a Perturbative Selection made Iteratively" (CIPSI)³⁴⁻³⁹ to obtain near full configuration interaction (FCI) quality VTEs for systems containing from 1 to 3 nonhydrogen atoms, coupled cluster (CC) with singles, doubles, triples and, quadruples (CCSDTQ)⁴⁰ for molecules encompassing 4 nonhydrogen atoms, and CC with singles, doubles, and triples (CCSDT)⁴¹⁻⁴⁵ for larger derivatives. Popular basis set correction schemes have often been applied. For example, the CCSDTQ VTEs computed with a double- ζ basis set were corrected thanks to CCSDT values obtained with a triple- ζ basis set. Most TBEs included in the QUEST database were estimated to be chemically accurate (corresponding to 1 kcal·mol⁻¹ or 0.04 eV error), with a typical error

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bar of ±0.03 eV. The reference values included in QUEST have been used by various groups, for example, to (i) assess the relative accuracies of third-order,⁴⁶ multireference,⁴⁷ and other emerging^{48–53} methods, (ii) quantify the accuracy of local hybrids for triplet ESs,⁵⁴ (iii) determine the relative performance of several hybrid⁵⁵ and double hybrid^{56–59} functionals, and (iv) evaluate the potential of orbital-optimized density functional theory for double excitations.^{60,61}

Despite burning an unreasonable number of CPU hours during the past 4 years, we could hardly "do better" than what is described above, as we rapidly hit the computational wall of both high-order CC schemes and/or large CIPSI calculations. For instance, CCSDTQ formally scales as $O(N^{10})$ (where N is the number of basis functions), and determining VTEs in molecules like furan or thiophene is nearly impossible even in a double- ζ basis set. In an effort to go one step further, we explored very recently, and for the first time, the performance of the approximate fourth-order CC model, CC4,⁶² in the context of ESs.⁶³ From a theoretical point of view, CC4 can be viewed as an approximation of CCSDTQ that still includes iterative quadruples, but neglects the calculation of the most expensive components (and avoids the storage of the higherexcitation amplitudes), allowing to reduce the formal scaling by 1 order of magnitude to $O(N^9)$.⁶² In our first investigation,⁶³ we considered very small systems (BH, BF, CO, HCl, H2O, H_2S , N_2 , and NH_3) for which well-converged CIPSI calculations were achievable for 25 ESs. This preliminary study indicates that CC4 indeed provides highly competitive VTEs, as it allows to significantly reduce the CCSDT error as compared to FCI, with final deviations-for ESs with a dominant single excitation character—only slightly larger than their CCSDTQ counterparts.

In the present contribution, which is, to our knowledge, only the second work presenting CC4 calculations for ESs, we build on our previous study,⁶³ and we aim at (i) evaluating the performance of CC4 for a more significant set of molecules and a larger variety of ESs, (ii) assessing the accuracy of additive basis set correction procedures based on CC4, and (iii) providing improved TBEs for many ESs included in the QUEST database.

2. COMPUTATIONAL DETAILS

All calculations presented here rely on the frozen-core approximation and the high-quality ground-state geometries extracted from the QUEST database.³³ For the sake of reproducibility, Cartesian coordinates for all systems displayed in Figure 1 are reproduced in the Supporting Information (SI). Note that, below, we do not specify the equation-of-motion (EOM) prefix for the CC calculations, although all ES calculations are performed using this formalism.

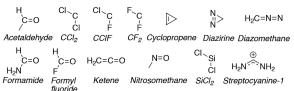
Most of our CC calculations have been performed with CFOUR,^{64,65} which provides an efficient implementation of high-order CC methods up to quadruples.^{66,67} For all considered systems, we performed, when technically achievable, CC2,^{68,69} CCSD,^{70–74} CC3,^{75–77} CCSDT,^{41–45} CC4,^{62,78} and CCSDTQ^{40,78} calculations using three Gaussian basis sets: 6-31+G(d), aug-cc-pVDZ, and aug-cc-pVTZ. For the sake of conciseness, these basis sets are sometimes dubbed as "Pop", "AVDZ", and "AVTZ" in the following. These three bases were selected because they contain both polarization and diffuse and are widely available, aug-cc-pVTZ being typically



=	—	H ₂ C=O	ĊI	т Г	≡P	P=O H	P=S H	
Acetylene	Ethylene	Formaldehyde	HCCI	HCF	HCP	HPO	HPS	
H Si F	H ₂ C=NH	N=O H	H ₂ C=S	Si	H ₂ C	=S		
HSiF	Methanimir	ne Nitroxvl	Silvlider	ne Ti	hioform	aldehvde		

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3 (non-H) atom molecules

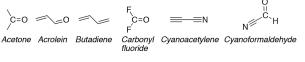


4 (non-H) atom molecules

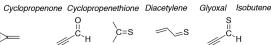
N=

Cyanogen

D



—≡N D=O D=S ≡



≡ 0=__0

Methylenecyclopropene Propynal Thioacetone Thioacrolein Thiopropynal

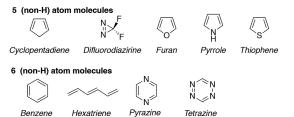


Figure 1. Representation of the systems investigated in the present study.

viewed as providing results close from the basis set limit for VTEs, except for very diffuse Rydberg states.

The identification of all states follows the QUEST database.³³ The separation between valence and Rydberg transitions was performed by examining the dominant unoccupied orbital contribution in the CCSD and CC3 results, which typically gives a clear trend, except for a few high-lying ESs for which mixing can appear. The double-excitation character of all transitions was probed by using $\%T_1$ the percentage of single excitations as given by CC3. Note that genuine or pure double ESs have a negligible $\%T_1$ and are unseen by CC2 and CCSD approaches.

In the statistical analysis presented below, we report the usual indicators: the mean signed error (MSE), the mean absolute error (MAE), as well as largest positive and negative deviations [Max(+) and Max(-), respectively].

CCSDTQP/6-31+G(d) calculations have been performed with MRCC^{79,80} for the molecules encompassing two nonhydrogen atoms (see Figure 1). These values are reported in Table S1 of the SI. Considering all 32 VTEs and taking these CCSDTQP values as references, we obtain a MAE of 0.003 eV for the CCSDTQ data determined with the same basis set. Unsurprisingly, the largest error, 0.023 eV, comes from the pure $(n, n) \rightarrow (\pi^*, \pi^*)$ doubly excited state of

Table 1. Statistical Analysis Obtained Using Various Correction Schemes^a

			107				6.15
approximating	using	and correcting with	MSE	MAE	Max(+)	Max(-)	useful?
CCSDTQ/aug-cc-pVTZ	CC4/aug-cc-pVTZ	no correction	0.002	0.003	0.012	-0.003	
		+ [CCSDTQ – CC4]/aug-cc-pVDZ	0.000	0.001	0.003	-0.003	22/31
		+ [CCSDTQ - CC4]/6-31+G(d)	0.001	0.002	0.006	-0.004	19/31
	CCSDT/aug-cc-pVTZ	no correction	-0.001	0.017	0.107	-0.049	
		+ [CCSDTQ - CCCST]/aug-cc-pVDZ	0.003	0.004	0.022	-0.004	27/31
		+ $[CCSDTQ - CCSDT]/6-31+G(d)$	0.001	0.003	0.030	-0.005	25/31
	CC3/aug-cc-pVTZ	no correction	0.006	0.017	0.124	-0.034	
		+ [CCSDTQ – CC3]/aug-cc-pVDZ	0.002	0.004	0.019	-0.003	28/31
		+ [CCSDTQ – CC3]/6-31+G(d)	0.001	0.004	0.020	-0.008	26/31
	CCSD/aug-cc-pVTZ	no correction	0.071	0.071	0.273	-0.005	
		+ [CCSDTQ – CCSD]/aug-cc-pVDZ	0.022	0.024	0.066	-0.011	26/31
		+ $[CCSDTQ - CCSD]/6-31+G(d)$	0.010	0.032	0.061	-0.042	25/31
	CC2/aug-cc-pVTZ	no correction	-0.008	0.215	0.348	-0.695	
		+ [CCSDTQ – CC2]/aug-cc-pVDZ	0.032	0.032	0.196	-0.001	29/31
		+ [CCSDTQ - CC2]/6-31+G(d)	-0.005	0.051	0.108	-0.353	29/31
CC4/aug-cc-pVTZ	CCSDT/aug-cc-pVTZ	no correction	0.004	0.016	0.095	-0.055	
		+ [CC4 – CCSDT]/aug-cc-pVDZ	0.004	0.004	0.022	-0.005	52/59
		+ [CC4 - CCSDT]/6-31+G(d)	0.002	0.004	0.028	-0.009	55/59
	CC3/aug-cc-pVTZ	no correction	0.006	0.016	0.112	-0.043	
	0	+ [CC4 - CC3]/aug-cc-pVDZ	0.002	0.003	0.019	-0.004	52/59
		+ [CC4 - CC3]/6-31+G(d)	0.000	0.004	0.018	-0.010	52/59
	CCSD/aug-cc-pVTZ	no correction	0.083	0.084	0.261	-0.009	
	0	+ [CC4 – CCSD]/aug-cc-pVDZ	0.018	0.019	0.063	-0.009	54/59
		+ [CC4 - CCSD]/6-31+G(d)	0.008	0.023	0.081	-0.041	51/59
	CC2/aug-cc-pVTZ	no correction	0.053	0.193	0.427	-0.701	
	0 1	+ [CC4 - CC2]/aug-cc-pVDZ	0.027	0.029	0.196	-0.016	56/59
		+ [CC4 - CC2]/6-31+G(d)	0.000	0.049	0.183	-0.352	54/59
CCSDT/aug-cc-pVTZ	CC3/aug-cc-pVTZ	no correction	0.004	0.015	0.073	-0.046	
0 1	0 1	+ [CCSDT – CC3]/aug-cc-pVDZ	-0.002	0.003	0.007	-0.016	109/118
		+ $[CCSDT - CC3]/6-31+G(d)$	-0.002	0.005	0.011	-0.036	102/118
	CCSD/aug-cc-pVTZ	no correction	0.114	0.114	0.539	0.010	,
	, 0 1	+ [CCSDT - CCSD]/aug-cc-pVDZ	0.022	0.025	0.073	-0.015	115/118
		+ $[CCSDT - CCSD]/6-31+G(d)$	0.012	0.027	0.071	-0.044	112/118
	CC2/aug-cc-pVTZ	no correction	-0.007	0.191	0.495	-0.675	
	-,	+ [CCSDT – CC2]/aug-cc-pVDZ	0.020	0.023	0.174	-0.020	115/118
		+ $[CCSDT - CC2]/6-31+G(d)$	-0.019	0.047	0.167	-0.355	106/118
			.1 1		1.6	1 555	100,110

"MSE, MAE, and maximal deviations (in eV) are obtained with respect to the reference method given in the leftmost column. The number of reference ESs considered at the CCSDTQ/aug-cc-pVTZ, CC4/aug-cc-pVTZ, and CCSDT/aug-cc-pVTZ levels is 31, 59, and 118, respectively. In the rightmost column, we provide the number of states for which the correction provides smaller (or equal) absolute errors as compared to the noncorrected scheme (rows labeled as "no correction").

nitroxyl for which the CC expansion obviously converges slower. Removing this pathological case yields a MAE of 0.002 eV with all errors below 0.010 eV, clearly confirming the quality of the CCSDTQ estimates.

3. METHODOLOGICAL ASPECTS

3.1. How Does CC4 Compare to CCSDTQ? Given that CC4 is an approximation of CCSDTQ and that the accuracy of the latter is recognized as exceptional for ESs with a dominant contribution from singly excited determinants (see above), it seems natural to assess the performance of CC4 with respect to CCSDTQ. Considering the values listed in the SI (with the three basis sets and all compounds), we have 220 CCSDTQ reference values at hand. Using these reference values, one obtains a MSE of 0.002 eV and a MAE as small as 0.003 eV for CC4. As one could have foreseen,⁶³ the largest deviations between CC4 and CCSDTQ originate from the genuine double excitations in nitroxyl, nitrosomethane, and glyoxal for which CC-based methods would likely not be chosen in

practical applications. Removing these three states, but keeping all other cases including the $2^{1}A_{g}$ dark state of butadiene, leads to a MSE of 0.001 eV and a MAE of 0.003 eV for CC4. Interestingly, in our earlier work devoted to very small compounds, we obtained the same MAE of 0.003 eV for CC4 (against CCSDTQP).⁶³ Clearly, one can state that CC4 is a very good approximation of CCSDTQ for the ESs that are not dominated by doubly excited determinants.

3.2. Is CC4 Significantly More Accurate Than CCSDT and CC3? The next natural question is to determine if CC4 is worth its cost. Indeed, CC4 formally scales as $O(N^9)$ which is more costly than both CCSDT and CC3 [which scale as $O(N^8)$ and $O(N^7)$, respectively]. Using again the available CCSDTQ values as references, we obtain MSE and MAE of 0.000 and 0.015 eV for CCSDT and 0.006 and 0.018 eV for CC3, again discarding from the set the true double excitations of nitroxyl, nitrosomethane, and glyoxal. It is no surprise that, on the one hand, both CC3 and CCSDT deliver results that would be rated as accurate enough for many applications, and, on the other hand, CC3 is an excellent approximation of CCSDT.^{33,81,82} The most valuable observation for our purposes is that CC4 indeed lowers the CCSDT and CC3 deviations by a factor of 5, which we consider a very significant improvement when one aims at producing highly accurate reference values.

3.3. Are CC4 Basis Set Effects Transferable? When defining values for benchmarking purposes, it is obvious from the discussion above that one can safely employ CC4/aug-cc-pVTZ as reference when neither well-converged CIPSI/aug-cc-pVTZ nor CCSDTQ/aug-cc-pVTZ calculations are technically achievable. As detailed below, this is the case, for example, for some of the molecules encompassing four nonhydrogen atoms. However, as customary in the field, ^{31,32,56,78,83–89} one can employ a double- ζ VTE computed with a CC model including quadruples in order to correct a triple- ζ VTE obtained with a quadruple-free CC method.

To test such strategies, we report in Table 1 the statistical analysis obtained with various correction schemes. We consider as reference the actual results obtained with the target method (leftmost column of Table 1). These reference values are available in Tables S3, S6, and S9 of the SI. In the rightmost column, we provide the number of states for which the correction yields smaller (or equal) absolute errors as compared to the noncorrected scheme (rows labeled as "no correction").

Globally, the data listed in Table 1 are quite appealing and one notices three general trends. First, adding corrections for higher-order excitations systematically decrease the MAE and improves the (vast) majority of the estimates. For example, while CC2/aug-cc-pVTZ returns a MAE of 0.215 eV as compared to CCSDTQ/aug-cc-pVTZ, correcting the former values with the differences between the VTEs computed with these two models but with the much smaller 6-31+G(d) basis set allows reducing the error to 0.051 eV and improves 29 of the 31 VTEs. Second, the more refined the starting point, the more accurate the final estimate. For example, using the same approach as above but starting from CC3/aug-cc-pVTZ rather than CC2/aug-cc-pVTZ would cut down the MAE from 0.051 to 0.004 eV. Third, performing the correction with aug-ccpVDZ typically yields smaller statistical deviations than with 6-31+G(d), though the difference between the two approaches is small

Given that CC4 is such a stunning approximation of CCSDTQ for ESs with a single excitation character, it is particularly relevant to investigate CC4/aug-cc-pVTZ estimates based on CCSDT and CC3, i.e.:

$$\Delta \tilde{E}_{AVTZ}^{CC4} \simeq \Delta E_{AVTZ}^{CCSDT} + \left[\Delta E_{AVDZ}^{CC4} - \Delta E_{AVDZ}^{CCSDT}\right]$$
(1a)

$$\Delta \tilde{E}_{AVTZ}^{CC4} \simeq \Delta E_{AVTZ}^{CCSDT} + \left[\Delta E_{Pop}^{CC4} - \Delta E_{Pop}^{CCSDT}\right]$$
(1b)

$$\Delta \tilde{E}_{AVTZ}^{CC4} \simeq \Delta E_{AVTZ}^{CC3} + \left[\Delta E_{AVDZ}^{CC4} - \Delta E_{AVDZ}^{CC3}\right]$$
(1c)

$$\Delta \tilde{E}_{AVTZ}^{CC4} \simeq \Delta E_{AVTZ}^{CC3} + \left[\Delta E_{Pop}^{CC4} - \Delta E_{Pop}^{CC3}\right]$$
(1d)

Looking at Table 1, one notices that all four approaches deliver very comparable error patterns, with MAE not larger than 0.004 eV, and global improvement over the uncorrected CCSDT/aug-cc-pVTZ and CC3/aug-cc-pVTZ data, the errors being cut down by a factor of 3. Of course, as can be deduced from the last column, not all values are more accurate when

adding corrections obtained with smaller basis sets, but improvements are observed for the vast majority of the cases.

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Additionally, although this is not the focus of the present work, we stress that the following expressions:

$$\Delta \tilde{E}_{\text{AVTZ}}^{\text{CCSDT}} \simeq \Delta E_{\text{AVTZ}}^{\text{CC3}} + \left[\Delta E_{\text{AVDZ}}^{\text{CCSDT}} - \Delta E_{\text{AVDZ}}^{\text{CC3}}\right]$$
(2a)

$$\Delta \tilde{E}_{\text{AVTZ}}^{\text{CCSDT}} \simeq \Delta E_{\text{AVTZ}}^{\text{CC3}} + \left[\Delta E_{\text{Pop}}^{\text{CC3DT}} - \Delta E_{\text{Pop}}^{\text{CC3}}\right]$$
(2b)

are extremely effective with MAE of 0.003 and 0.005 eV and small maximal deviations as compared to the true CCSDT/ aug-cc-pVTZ transition energies. The fact that these two approximations are very accurate is certainly valuable, as both CC3/aug-cc-pVTZ and CCSDT/6-31+G(d) calculations are technically achievable on systems with up to approximately 10-12 nonhydrogen atoms, allowing to significantly expand the number of TBEs based on CCSDT/aug-cc-pVTZ that are included in the QUEST database for larger systems.

The take-home message of this section is that, while these correction schemes (which are widespread in the CC community) are generally valuable to produce accurate VTEs, their overall accuracy depends ultimately on the quality of the starting point.

4. REFINED THEORETICAL BEST ESTIMATES

Given the above observations, it is worth reexamining our previous TBEs with the additional accuracy provided by CC4.

4.1. Linear Systems. The QUEST database includes three linear systems encompassing four nonhydrogen atoms: cyanoacetylene, cyanogen, and diacetylene.^{11,33} For these three systems CC4/aug-cc-pVTZ calculations could be performed, whereas CCSDTQ/aug-cc-pVTZ calculations remain beyond reach (see Table S9 in the SI for raw data). We have, therefore, computed new TBEs on the basis of these CC4 data, applying the very trustworthy basis set correction (see top lines of Table 1).

$$\Delta E_{\text{AVTZ}}^{\text{TBE}} = \Delta E_{\text{AVTZ}}^{\text{CC4}} + \left[\Delta E_{\text{AVDZ}}^{\text{CCSDTQ}} - \Delta E_{\text{AVDZ}}^{\text{CC4}}\right]$$
(3)

The results for seven ESs of these three linear molecules are listed in Table 2. It can be seen that the variations are negligible, with three TBEs unchanged and four decreasing by 0.01 eV.

4.2. Four Nonhydrogen Compounds. For most nonlinear compounds encompassing four nonhydrogen nuclei, the TBEs of the QUEST database were obtained with CCSDT/

Table 2. TBE/aug-cc-pVTZ (in eV) Established for the Linear Systems Encompassing Four Nonhydrogen Atoms^a

		TBEs			
molecule	transition	nature	new	prev.	diff.
cyanoacetylene	${}^{1}\Sigma^{-} (\pi \rightarrow \pi^{*})$	Val	5.79	5.80	-0.01
	$^{1}\Delta \ (\pi \rightarrow \pi^{*})$	Val	6.07	6.07	0.00
cyanogen	${}^{1}\Sigma_{u}^{-} (\pi \rightarrow \pi^{*})$	Val	6.38	6.39	-0.01
	$^{1}\Delta_{u} (\pi \rightarrow \pi^{*})$	Val	6.65	6.66	-0.01
	${}^{1}\Sigma_{u}^{-} (\pi \rightarrow \pi^{*})[\mathrm{F}]^{b}$	Val	5.04	5.05	-0.01
diacetylene	${}^{1}\Sigma_{u}^{-} (\pi \rightarrow \pi^{*})$	Val	5.33	5.33	0.00
	$^{1}\Delta_{u} (\pi \rightarrow \pi^{*})$	Val	5.61	5.61	0.00

^{*a*"}Val" stands for valence ESs. TBEs obtained with eq 3 using data from the SI. The previous TBEs taken from ref 11 are given for comparison. ^{*b*}Fluorescence from the optimized excited-state geometry.

aug-cc-pVTZ corrected using CCSDTQ/6-31+G(d) to take into account the quadruples contribution.^{11,33} Given the results gathered in Table 1, it is not clear that applying CC4/aug-cc-pVDZ corrections would improve significantly these original estimates. However, for a few compounds, the QUEST TBEs do not include corrections from the quadruples, as they were computed at the CCSDT/aug-cc-pVTZ level "only". Hence, we have considered these derivatives and employed eq 1a to determine new TBEs. The results are listed in Table 3 and compared to the previous estimates.

Table 3. TBE/aug-cc-pVTZ (in eV) Established for Nonlinear Systems Containing Four Nonhydrogen Atoms Obtained Using Eq $1a^a$

		TBEs			
molecule	transition	nature	new	prev.	diff.
acrolein	${}^{1}A'' \ (n \rightarrow \pi^*)$	Val	3.72	3.78 ^b	-0.06
	$^{1}A' (\pi \rightarrow \pi^{*})$	Val	6.67	6.69	-0.02
	${}^{1}A'' \ (n \rightarrow \pi^*)$	Val	6.69	6.72 ^c	-0.03
	${}^{1}A' (n \rightarrow 3s)$	Ryd	7.11	7.08	+0.03
	${}^{1}A' \ (\pi \rightarrow \pi^{*})$	Val ^d	7.93	7.87 ^b	+0.06
carbonylfluoride	${}^{1}A_{2} (n \rightarrow \pi^{*})$	Val	7.29	7.31 ^e	-0.02
cyanoformaldehyde	${}^{1}A'' \ (n \rightarrow \pi^*)$	Val	3.82	3.81	+0.01
	${}^{1}A'' \ (\pi \rightarrow \pi^*)$	Val	6.43	6.46	-0.03
isobutene	${}^{1}B_{1} (\pi \rightarrow 3s)$	Ryd	6.48	6.46	+0.02
	${}^{1}A_{1} (\pi \rightarrow 3p)$	Ryd	7.01	7.01	0.00
propynal	${}^{1}A'' \ (n \rightarrow \pi^*)$	Val	3.81	3.80	+0.01
	${}^{1}A'' \ (\pi \rightarrow \pi^*)$	Val	5.51	5.54	-0.03
thioacrolein	${}^{1}A'' \ (n \rightarrow \pi^*)$	Val	2.10	2.11	-0.01
thiopropynal	${}^{1}A'' \ (n \rightarrow \pi^*)$	Val	2.02	2.03	-0.01

^a"Val" and "Ryd" stand for valence and Rydberg ESs. Raw data are given in the SI. The previous TBE/aug-cc-pVTZ values taken from refs 11, 29, 30, and 33 are given for comparison. ^bTBEs obtained from a basis set corrected CIPSI/6-31+G(d) estimate. The CCSDT/augcc-pVTZ values for these two specific states are 3.73 and 8.01 eV. ^cConsidered as *unsafe* in ref 11. ^dES with a significant double excitation character. ^eTBE obtained from a basis set corrected CIPSI/ 6-31+G(d) estimate.

Although the differences listed in the rightmost column of Table 3 have different signs, the impact of CC4 is typically a small decrease of the previous estimates, hinting that CCSDT tends to overestimate the VTEs. This is consistent with the 0.003 eV MSE reported in Table 1 for CCSDT as compared to CC4. For the 13 states listed in Table 3, the average correction is -0.011 eV and the average absolute change is 0.022 eV only. More importantly, for the vast majority of cases, the changes are not larger than ± 0.03 eV, which was the estimated error bar in ref 11.

Nevertheless, a molecule worth discussing is acrolein. For its lowest hallmark $n \rightarrow \pi^*$ transition, the CCSD, CC3, and CCSDT VTEs obtained with aug-cc-pVTZ are 3.913, 3.743, and 3.725 eV. The new TBE, 3.72 eV, is consistent with this trend, but is significantly smaller than the original CIPSI-based estimate of 3.78 eV. It is likely that the CIPSI extrapolation error bar was underestimated previously, and we believe that our current TBE is more accurate. Indeed, CC4/6-31+G(d) and CCSDTQ/6-31+G(d) values are very consistent (see Table S7). The second ES, a strongly dipole-allowed ¹A' ($\pi \rightarrow \pi^*$) transition, was slightly blueshifted in our original work, but the error, -0.02 eV, remains low, a statement also holding for

the higher-lying Rydberg ES of the same symmetry. The original estimate for the second ${}^{1}A''$ $(n \rightarrow \pi^{*})$ transition, 6.72 eV, was labeled as unsafe in ref 11 due to its significant double excitation character. Indeed, its percentage of single excitations $(\%T_1)$ is only 79.4% according to CC3/aug-cc-pVTZ. Nevertheless, the CC4 correction obtained with aug-ccpVDZ is not very large, and one can likely claim that the new TBE of 6.69 eV stands as the most trustworthy estimate published to date for this particular ES. The higher-lying ${}^{1}A'$ (π $\rightarrow \pi^*$) transition of acrolein has a nature similar to the famous A_{σ} state of butadiene with a % T_1 of 75%. At the CIPSI/6- $3\hat{1}+G(d)$ level, a value of 8.00 \pm 0.03 eV was obtained previously,²⁹ and our CC4 value obtained with the same basis set is consistent with this result: 8.035 eV (see Table S7 in the SI). Based on CC4/aug-cc-pVDZ, i.e., using eq 1a the new TBE is 7.93 eV, upshifted by 0.06 eV as compared to the original one. Given that for the similar transition in butadiene, the changes between CC4 and CCSDTQ are trifling (Table S7), we consider this new TBE as more trustworthy than the original one, though giving a reliable error bar is not straightforward. Finally, one can also note that, for carbonylfluoride, the current TBE is 0.02 eV smaller than the original CIPSI-based one, for which the estimated extrapolation error bar was 0.02 eV.¹¹

4.3. Larger Systems. For the largest systems of Figure 1, CC4 has the advantage to provide a systematic path toward high-accuracy for systems that are beyond reach at the CCSDTQ level. Let us first illustrate this with a set of 30 ESs extracted from systems with five (nonhydrogen) atom (Table 4). To the best of our knowledge, for all five molecules, our results provide the first VTEs including quadruples. As one can see, the differences with the previous TBEs are small and are typically slightly negative, confirming the outcome of the previous section showing that CCSDT/aug-cc-pVTZ yields slightly too large VTEs. The average absolute correction for the data of Table 4 is nonetheless very small, 0.02 eV, which confirms that quality of the original TBEs from the QUEST database.

Let us now take a look at the most problematic cases. Unsurprisingly, the ${}^{1}A_{1}$ ($\pi \rightarrow \pi^{*}$) transition of cyclopentadiene is significantly red-shifted by the CC4 correction (-0.07 eV). This transition was previously classified as *unsafe*,¹¹ due to its ${}^{\otimes}T_{1}$ value of 78.9% and its similar nature to the famous ${}^{1}A_{g}$ ES of butadiene. For the latter compound, the difference between the CIPSI/6-31+G(d) and CCSDT/6-31+G(d) estimates is -0.08 eV,²⁹ which suggests that the CC4 correction of -0.07 eV for the ${}^{1}A_{1}$ ES of cyclopentadiene is very reasonable. The lowest B_{2} ES of thiophene is subject to a -0.04 eV shift despite having a very large single excitation character (${}^{\otimes}T_{1} = 91.5{}^{\otimes}$). All other ESs are even less affected by the CC4 corrections with variations of ± 0.03 eV at most.

We now consider three highly symmetric six-membered rings, namely benzene, pyrazine, and tetrazine, for which CC4/6-31+G(d) calculations are still doable. The results obtained for a large number of ESs are listed in Table 5. For these three compounds, this table is, as far as we are aware of, the first to propose CC-based VTEs including corrections from iterative quadruples.

For benzene, we consider three valence and four Rydberg ESs. These ESs have a very strong single excitation character with $\%T_1 > 92\%$, except for the lowest ${}^1B_{2u}$ transition ($\%T_1 = 86\%$). In all cases, one finds that CC4/6-31+G(d) provides

Table 4. TBE/aug-cc-pVTZ (in eV) Established for Systems Containing Five Nonhydrogen Atoms^a

		TBEs			
molecule	transition	nature	new	prev.	diff.
cyclopentadiene	${}^{1}B_{2} (\pi \rightarrow \pi^{*})$	Val	5.54	5.56 ^b	-0.02
	${}^{1}A_{2} (\pi \rightarrow 3s)$	Ryd	5.78	5.78	0.00
	${}^{1}B_{1} (\pi \rightarrow 3p)$	Ryd	6.41	6.41	0.00
	${}^{1}A_{2} (\pi \rightarrow 3p)$	Ryd	6.45	6.46	-0.01
	${}^{1}B_{2} (\pi \rightarrow 3p)$	Ryd	6.56	6.56	0.00
	${}^{1}A_{1} (\pi \rightarrow \pi^{*})$	Val	6.45	6.52 ^c	-0.07
difluorodiazirine	${}^{1}B_{1} (n \rightarrow \pi^{*})$	Val	3.73	3.74	-0.01
	$^{1}A_{2} (\pi \rightarrow \pi^{*})$	Val	6.99	7.00	-0.01
	${}^{1}B_{2} (n.d.)^{d}$	Ryd	8.50	8.52	-0.02
furan	${}^{1}A_{2} (\pi \rightarrow 3s)$	Ryd	6.10	6.09	+0.01
	${}^{1}B_{2} (\pi \rightarrow \pi^{*})$	Val	6.35	6.37	-0.02
	$^{1}A_{1} (\pi \rightarrow \pi^{*})$	Val	6.53	6.56	-0.03
	${}^{1}B_{1} (\pi \rightarrow 3p)$	Ryd	6.65	6.64	+0.01
	${}^{1}A_{2} (\pi \rightarrow 3p)$	Ryd	6.82	6.81	+0.01
	${}^{1}B_{2} (\pi \rightarrow 3p)$	Ryd	7.25	7.24	+0.01
pyrrole	${}^{1}A_{2} (\pi \rightarrow 3s)$	Ryd	5.23	5.24	-0.01
	${}^{1}B_{1}$ (n.d.)	Ryd	5.97	6.00	-0.03
	${}^{1}A_{2} (\pi \rightarrow 3p)$	Ryd	6.01	6.00	+0.01
	${}^{1}B_{1}$ (n.d.) ^{<i>e</i>}	Ryd	6.09		
	${}^{1}B_{2} (\pi \rightarrow \pi^{*})$	Val	6.24	6.26	-0.02
	$^{1}A_{1} (\pi \rightarrow \pi^{*})$	Val	6.27	6.30	-0.03
	${}^{1}B_{2} (\pi \rightarrow 3p)$	Ryd	6.82	6.83	-0.01
thiopehene	$^{1}A_{1} (\pi \rightarrow \pi^{*})$	Val	5.62	5.64	-0.02
	${}^{1}B_{2} (\pi \rightarrow \pi^{*})$	Val	5.94	5.98	-0.04
	${}^{1}A_{2} (\pi \rightarrow 3s)$	Ryd	6.13	6.14	-0.01
	$^{1}B_{1} (\pi \rightarrow 3p)$	Ryd	6.12	6.14	-0.02
	${}^{1}A_{2} (\pi \rightarrow 3p)$	Ryd	6.23	6.21	+0.02
	${}^{1}B_{1} (\pi \rightarrow 3s)$	Ryd	6.49	6.49	0.00
	${}^{1}B_2 \ (\pi \rightarrow 3p)^f$	Ryd	7.27	7.29	-0.02
_	$^{1}A_{1} (\pi \rightarrow \pi^{*})^{f}$	Val	7.30	7.31 [°]	-0.01

^{*a*"}Val" and "Ryd" stand for valence and Rydberg ESs. TBEs established with eq 1b, except for difluorodiazirine and furan for which eq 1a is used. Raw data are given in Table S10 in the SI. The previous TBEs taken from refs 11 and 30 are also listed. ^{*b*}The most recent TBE, based on a basis set corrected CIPSI/6-31+G(d) value, is 5.54 eV; see ref 33. ^{*c*}Considered as *unsafe* in ref 11. ^{*d*}Incorrectly labeled as valence in ref 30. ^{*e*}Not considered previously. ^{*f*}Nonnegligible valence/Rydberg mixing.

small negative or null corrections, with a maximal amplitude of -0.02 eV, even for the lowest-energy transition.

Eleven ESs of pyrazine are listed in Table 5, including four Rydberg states. The corrections to previous CCSDT-based TBEs are either positive, null, or negative, but most are within the expected error bar of the original estimates. Nevertheless, for the lowest $1^{1}B_{1g}$ ES, corresponding to a $n \rightarrow \pi^{*}$ excitation, a relatively large correction of -0.04 eV is noticed. This transition has a rather low $\%T_1$ value of 84%, which however does not translate into a large CC3/CCSDT difference (ca. 0.01 eV for the three basis sets, see the SI). Likewise the highest-lying $1^{1}B_{1u}$ ES also undergoes a -0.04 eV correction, but this state was rated *unsafe* previously,^{11,33} due to an unusually large CC3/CCSDT difference. Four other ESs have their VTEs corrected by $\pm 0.03 \text{ eV}$ by CC4, a shift corresponding to the expected error bar of the original TBEs.

In tetrazine, we consider 12 ESs, including two with a pure double-excitation character, for which NEVPT2 was employed pubs.acs.org/JCTC

Table 5. TBE/aug-cc-pVTZ (in eV) Established for Benzene, Pyrazine, and Tetrazine^a

			TBEs				
molecule	transition	nature	new	prev.	diff.		
benzene	$^{1}B_{2u} (\pi \rightarrow \pi^{*})$	Val	5.05	5.06	-0.0		
	${}^{1}B_{1u} (\pi \rightarrow \pi^{*})$	Val	6.43	6.45	-0.02		
	${}^{1}E_{1g} (\pi \rightarrow 3s)$	Ryd	6.52	6.52	0.0		
	$^{1}A_{2u} (\pi \rightarrow 3p)$	Ryd	7.08	7.08	0.0		
	${}^{1}E_{2u} (\pi \rightarrow 3p)$	Ryd	7.15	7.15	0.0		
	${}^{1}A_{1u} (\pi \to 3p)^{b}$	Ryd	7.23				
	$^{1}E_{1u} (\pi \rightarrow \pi^{*})^{b}$	Val	7.17				
pyrazine	${}^{1}B_{3u} (n \rightarrow \pi^{*})$	Val	4.14	4.15	-0.0		
	${}^{1}A_{u} (n \rightarrow \pi^{*})$	Val	4.97	4.98	-0.0		
	${}^{1}B_{2u} (\pi \rightarrow \pi^{*})$	Val	4.99	5.02	-0.0		
	${}^{1}B_{2g} (n \rightarrow \pi^{*})$	Val	5.68	5.71	-0.0		
	${}^{1}A_{g}(n \rightarrow 3s)$	Ryd	6.66	6.65	+0.0		
	${}^{1}B_{1g}(n \rightarrow \pi^{*})$	Val	6.70	6.74	-0.0		
	$^{1}B_{1u} (\pi \rightarrow \pi^{*})$	Val	6.85	6.88	-0.0		
	${}^{1}B_{1g} (\pi \rightarrow 3s)$	Ryd	7.20	7.21	-0.0		
	${}^{1}B_{2u} (n \rightarrow 3p)$	Ryd	7.27	7.24	+0.0		
	${}^{1}B_{1u} (n \rightarrow 3p)$	Ryd	7.45	7.44	+0.0		
	${}^{1}B_{1u} (\pi \rightarrow \pi^{*})$	Val	7.94	7.98 ^c	-0.0		
tetrazine	${}^{1}B_{3u} (n \rightarrow \pi^{*})$	Val	2.46	2.47	-0.0		
	${}^{1}A_{u} (n \rightarrow \pi^{*})$	Val	3.68	3.69	-0.0		
	${}^{1}A_{g} (n, n \rightarrow \pi^{*}, \pi^{*})$	Val		4.61 ^d			
	${}^{1}B_{1g} (n \rightarrow \pi^*)$	Val	4.87	4.93	-0.0		
	$^{1}B_{2u} (\pi \rightarrow \pi^{*})$	Val	5.17	5.21	-0.0		
	${}^{1}B_{2g} (n \rightarrow \pi^{*})$	Val	5.50	5.45	-0.0		
	${}^{1}A_{u} (n \rightarrow \pi^{*})$	Val	5.51	5.53	-0.0		
	${}^{1}B_{3g}(n, n \to \pi^{*}, \pi^{*})$	Val		6.15 ^d			
	${}^{1}B_{2g}(n \rightarrow \pi^{*})$	Val	6.05	6.12	-0.0		
	${}^{1}B_{3g} (n \rightarrow 3s)^{b}$	Ryd	6.47				
	$^{1}B_{3u} (\pi \rightarrow \pi^{*})^{b}$	Val	6.67				
	${}^{1}B_{1g} (n \rightarrow \pi^{*})$	Val	6.89	6.91	-0.0		
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^{*a*"Val" and "Ryd" stand for valence and Rydberg ESs. TBEs established using eq 1b. Raw data are given in Table S11 in the SI. The previous TBEs taken from ref 11 are also listed. ^{*b*}Not considered previously. ^{*c*}Considered as *unsafe* in ref 11. ^{*d*}Genuine doubly excited states with TBEs obtained at the NEVPT2/aug-cc-pVTZ level and considered as *unsafe* in ref 11.}

in the QUEST database to obtain TBEs, whereas, for the other transitions, CCSDT/aug-cc-pVTZ or basis set corrected CCSDT/aug-cc-pVDZ values were originally selected as TBEs.¹¹ All the latter were rated as *safe* as the energy differences between CCSDT and CC3 were smaller than 0.03 eV despite $\%T_1$ values often smaller than 90%.^{11,33} For several ESs, small negative CC4 corrections, -0.01 or -0.02 eV, are obtained. For the lowest $\pi \to \pi^*$ transition, the correction is only slightly larger, i.e., - 0.04 eV. However, for three $n \rightarrow \pi^*$ transitions $(1^{1}B_{1g}, 1^{1}B_{2g})$ and $2^{1}B_{2g})$, much larger changes (-0.06, -0.05, and -0.07 eV) are induced by the inclusion of quadruples, indicating that our original assessment of the TBE quality was optimistic. Interestingly, for these three transitions, one has $\%T_1 < 85\%$, in contrast with the other singly excited ESs for which $\%T_1 > 85\%$. This observation suggests that the 85% barrier might be the limit for which CCSDT can be considered trustworthy, irrespective of the difference between CC3 and CCSDT. For the two doubly excited states of tetrazine listed in Tables 5 and S11 that are both characterized

			·	ГBEs	
molecule	transition	nature	new	prev.	diff.
hexatriene	${}^{1}B_{u} (\pi \rightarrow \pi^{*})$	Val	5.34	5.37	-0.03
	${}^{1}A_{g} (\pi \rightarrow \pi^{*})$	Val	5.46/5.43 ^c	5.62 ^b	-0.16/-0.19
	${}^{1}A_{u} (\pi \rightarrow 3s)$	Ryd	5.79	5.79	0.00
	${}^{1}B_{g} (\pi \rightarrow 3p)$	Ryd	5.93	5.94	-0.01

Table 6. TBE/aug-cc-pVTZ (in eV) Established for Hexatriene^a

^{*a*} "Val" and "Ryd" stand for valence and Rydberg ESs. TBEs established using eq 1d. Raw data are given in Table S11 in the SI. The previous TBEs taken from ref 33 are also listed. ^{*b*} Considered as *unsafe* in ref 33. ^{*c*} See text.

by $\%T_1 < 1\%$,¹¹ the differences between CC4/6-31+G(d) and CCSDT/6-31+G(d) are very large, that is, $-0.79 \text{ eV}(A_g)$ and $-0.99 \text{ eV}(B_{3g})$. These values are nonetheless typical for these challenging ESs with a dominant contribution from the doubly excited determinants. Applying eq 1b delivers VTEs of 5.17 and 6.44 eV. These two results logically remain higher in energy than the corresponding NEVPT2/aug-cc-pVTZ values of 4.61 and 6.15 eV, which we consider as the most accurate TBEs available to date, though with an error bar of approximately $\pm 0.1 \text{ eV}$.

Finally, let us briefly discuss the challenging case of hexatriene (Table 6). The original TBEs for this small polyene have been obtained via eq 2a: 5.37, 5.62, 5.79, and 5.94 eV for the lowest ${}^{1}B_{\mu}$, ${}^{1}A_{g}$, ${}^{1}A_{\mu}$, and ${}^{1}B_{g}$ ESs, respectively.³³ All were considered safe, except the second one that has a very significant contribution from the double excitations with $%T_1$ = 65% (i.e., 10% less than in butadiene). Using eq 1d to include CC4 corrections leads to improved TBEs of 5.34, 5.46, 5.79, and 5.93 eV. Only the ${}^{1}A_{g}$ VTE undergoes a significant downshift (-0.16 eV) due to the quadruples. Logically, this 5.46 eV value can be considered as an upper bound, and we roughly approximate the exact value to be of the order of 5.43 eV. To support this crude estimate, we looked at the equivalent transition in the smaller polyene, butadiene. Indeed, in butadiene, the difference between the CC4/6-31+G(d) and FCI/6-31+G(d) values is -0.01 eV; the correction should be larger for hexatriene given its smaller $%T_1$ value. In addition, as can be seen in ref 29, the NEVPT2/6-31+G(d) value is 0.20 eV larger than the FCI/6-31+G(d) result for butadiene, and applying such rigid shift to NEVPT2/aug-cc-pVTZ VTE of hexatriene²⁹ would yield an estimate of 5.44 eV. Although it would not be suited to rate this new TBE/aug-cc-pVTZ of 5.43 eV as safe, it is likely one of the most accurate estimate published to date for this dark transition.

5. CONCLUSIONS AND OUTLOOK

Three main problems were tackled in the present study. Our first aim was to confirm that CC4 provides very accurate VTEs. To this end, we considered 43 molecules containing between two and four (nonhydrogen) atoms and computed their ESs with three Gaussian basis sets containing diffuse functions, i.e., 6-31+G(d), aug-cc-pVDZ, and aug-cc-pVTZ. We started by defining more than 200 CCSDTQ reference values. Excluding the few pathological pure doubly excited states of $(n, n) \rightarrow (\pi^*, \pi^*)$ nature but conserving all other transitions that have a nonnegligible double excitation character $(2^1A_g \text{ in butadiene}, 2^1A'' \text{ in acrolein, etc})$, we showed that CC4 is an excellent approximation of CCSDTQ with a mean absolute error of 0.003 eV. This is a totally negligible deviation for the vast majority of chemical studies (<0.1 kcal·mol⁻¹). We also

showed that neither CC3 nor CCSDT could deliver the same level of accuracy for the very same set of ESs.

Second, we investigated the performance of additive basis set correction schemes in the EOM-CC context using the same set of ESs as in the first part. It appeared that these popular correction strategies indeed statistically improve the quality of the VTEs. More interestingly, such basis set corrections were found particularly powerful when starting from CC3/aug-cc-pVTZ or CCSDT/aug-cc-pVTZ, as they allowed estimating the true CC4/aug-cc-pVTZ or CCSDTQ/aug-cc-pVTZ VTEs with average errors of approximately 0.005 eV. Of course, it is essential to confirm these trends for larger compounds, but the cost of CC4/aug-cc-pVTZ and/or CCSDTQ/aug-cc-pVTZ unfortunately prevents us from doing it as of today.

Finally, CC4 was employed to improve previous CCSDTbased TBEs of the QUEST database.³³ For benzene, cyclopentadiene, difluorodiazirine, furan, hexatriene, pyrazine, pyrrole, tetrazine, and thiophene, it was possible to perform CC4/6-31+G(d) and/or CC4/aug-cc-pVDZ calculations, whereas CCSDTQ remains beyond reach. The corrections obtained were, in most cases, smaller than the expected average error of the original TBEs (ca. ± 0.03 eV). Nevertheless, most corrections were null or negative, hinting at a slight overestimation trend in our original TBEs. In addition, some of the estimates originally viewed as unsafe can be now considered as trustworthy $(2^1A'')$ in acrolein, 2^1A_1 in cyclopentadiene, $3^{1}A_{1}$ in thiophene, etc.), whereas a few optimistic assessments have been revised, resulting in more accurate TBEs (in particular for the valence B_{1g} and B_{2g} ESs of tetrazine).

We are currently pursuing our efforts to improve the quality, size, and diversity of the QUEST database, so as to provide the most trustworthy reference values possible to the community. We plan to publish, within a year or two, an expanded and improved version of the QUEST database.

ASSOCIATED CONTENT

Supporting Information

The Supporting Information is available free of charge at https://pubs.acs.org/doi/10.1021/acs.jctc.2c00416.

Raw VTEs and geometries (PDF)

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Notes

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